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SELECTING THE MOST EFFECTIVE ALTERNATIVE OF WATERPROOFING MEMBRANES FOR MULTIFUNCTIONAL INVERTED FLAT ROOFS

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Abstract. There are many various methods, techniques and materials for constructing multifunctional inverted flat roofs now. However, in practice, the constructed roofs of this type often have various defects, whose elimination is rather costly. To solve this problem, the analysis of multifunctional inverted flat roofs has been performed and their advantages and disadvantages, as well as building defects and mistakes made in the course of construction, have been demonstrated. Damp-proofing materials available on the market for making waterproofing membranes for multifunctional inverted flat roofs have been evaluated. The problem of choosing the most effective membrane alternative for the investigated type of roofs is solved by analysing ten alternatives with the help of multi-criteria evaluation method, SAW and three game theory rules determined by Hurwicz, Laplace and Bayes, as well as LEVI 3.0 program. The analysis of the results has shown that the alternative No. 5 is the best membrane alternative among the ten considered options.

Keywords: multifunctional inverted flat roof, multi-criteria evaluation, the selection of the best waterproofing membrane alternative, MCDM (Multi Criteria Decision Making), SAW (Simple Additive Weighting), game theory.

Introduction

Under the conditions, when various social processes, such as urbanization, take place in the world, all urban areas should be effectively used. The need for plots in the most prestigious city areas has become a global problem. Therefore, the ways of rational use of the available urban areas are being sought (Malinauskas, Kalibatas 2005). In this environment, one of the effective structural elements is a multifunctional inverted flat roof, which helps to effectively use the area on the top of a building. Such roofs may be used for creating green zones, car parking lots, terraces, playgrounds, etc. on their surfaces. They can be classified based on their functions and other characteristics, while various design solutions, materials and technologies may be used for their construction (Migilinskas 2002). According to the Lithuanian building specification STR 2.05.02:2008 "Building constructions. Roofs" (2008), roofs should be able to withstand the effects of rain and sunrays. They should be also resistant to fire, mechanical forces, environmental and chemical effects and be environmentally friendly. Roofs can protect people and their property from unfavourable climatic conditions only when they are properly designed and constructed of properly selected materials.

However, in practice, the construction of such roofs is a complicated problem, therefore, various defects, whose elimination is rather costly, can be often found in the constructed roofs. The authors of the paper, aiming to contribute to the solution of this problem, present the analysis of the considered roofs, which shows their advantages and disadvantages, as well as building defects and mistakes made in the course of construction. The novelty of this research is using the multiple criteria evaluation method for selecting the most effective waterproofing membranes for multifunctional inverted flat roofs according to the users' needs and Lithuanian building specification.

Apart from the Introduction, the following sections are included: Section 1 presents the analysis of multifunctional inverted flat roofs, while Section 2 gives a survey of Multi Attribute Decision Making (MADM) methods and their use in construction, as well as a description of the complex MADM and game theory-based method for assessing the alternatives, which was used in the study. Section 3 describes the selection of an effective alternative based on the multiple criteria evaluation method, SAW, and the analysis of the alternatives performed using three game theory rules (Hurwicz, Laplace, Bayes) and LEVI 3.0 program. Section 4 provides the obtained results and discussion, while final Section presents the conclusions.

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1. Analysis of multifunctional inverted flat roofs

In the field of building structures, a roofing system is a relevant part of the construction of a building – it affects the building's service life, durability and efficiency (Petrakova *et al.* 2004). Multifunctional flat roofs can be divided into traditional (ordinary) and inverted flat roof types, based on the location of a waterproofing membrane in their constructions (ST 121895674.215.01:2012 "Roof construction"). In standard (traditional) multifunctional roofs, a thermal insulating layer is found under a waterproofing layer (a damp course). In the inverted multifunctional roofs, a thermal insulating layer is arranged above the waterproofing layer of a roof.

Multifunctional inverted flat roofs appeared in the USA in the 50s of the 20th century, together with the invention of the extruded polystyrene foam (XPS). In Central Europe, roofs of this type came into use later, in the 70s of the 20th century (Kutnar 2005). In foreign literature, some other names of the inverted roofs, such as protected membrane or ballasted roofs (roofs with ballast intended for the cases when washed gravel, crushed stone or other concrete paving components are used for the upper layer) can be found.

A roofing system should satisfy several requirements to achieve its purpose from the viewpoint of its construction, aesthetics, materials and firmness of the roof structure, implying that it should have the utility value. In addition to the waterproof properties of the roofing material, roof envelopes should also have thermal characteristics determining the energy saving aspect of their appraised utility value. Last but not least, the roofing system also has to meet the requirements concerning the amount of money invested and the operating costs (costs for maintenance and repairs during its lifetime) (Petrakova *et al.* 2004).

According to Deal (1979), multifunctional inverted flat roofs have three main advantages due to the reverse order of their structural layers' arrangement, which are as follows:

- a large ballast (the upper pavement) layer and a thermal insulating layer ensure the complete protection of the damp course of the roof from the ultraviolet rays;
- the damp course is protected from physical and mechanical injuries;
- the number of temperature cycles is decreased and temperature shock possibility is eliminated in such roofs.

Slanina and Šilarova (2009) mentioned another advantage of multifunctional inverted roofs. A waterproofing membrane is placed on a supporting structure or on a slope layer and below a thermal insulation layer. This order of layers optimally solves the problem with moisture control of compact membrane roofs. The fundamental principle of moisture control, implying that diffusion resistance of each layer decreases according to a thermal gradient (usually, from the interior to the exterior surface of the assembly), is satisfied.

The applicability of multifunctional inverted flat roofs has been acknowledged in many countries of the world. Research institutes and regulating institutions in the European states have long ago approved using the extruded polystyrene foam in multifunctional inverted roof systems. The expanding range of XPS products stimulated the development of new types of the inverted flat roofs, such as a roof garden, a roof with parking lots, etc. (Cziesielski *et al.* 2001).

Vaitkus *et al.* (2006) state that operational characteristics of expanded polystyrene products strongly depend on their compressive strength, while this parameter is mostly determined by the structure and form of the granules of the material. Expanded polystyrene is deformed under load, but when the load is removed, the deformation decreases or even disappears. It has been found that its density depends on the average diameter of the granules.

González *et al.* (2014) have studied the possibilities and advantages offered by multifunctional inverted flat roofs. According to them, the heating system used in these roofs for more than 35 years in Europe proved to be very effective. The durability of the materials used and their performance in the structures have been studied for a long period of their service by the researchers from various institutes and by independent experts. The service life of the damp course of the considered roofs is much longer than that of ordinary roofs because it is completely protected from the exterior environmental effects and temperature variations.

However, multifunctional inverted flat roofs, like any other structural elements or materials, also have some drawbacks. Baskaran *et al.* (1997) described these drawbacks as follows:

- thermal insulation is exposed to water from rain and snow;
- ballast, increasing the load on a structure, must be used;
- these roofs cannot be used for all types of buildings;
- ballast may be blown away, possibly, exposing the insulation to UV;
- a membrane is difficult to inspect;
- these roofs are more expensive than conventional flat roofs.

According to Tobiasson (1994), a higher cost of the roofs can be attributed to using expensive extruded polystyrene foam (XPS) and ballast (for making the surface layer of the roof).

Multilayer systems of multifunctional inverted flat roofs (including the inverted roof gardens and traditional roofs) consist of the same elements, with the only difference being the arrangement order of roof layers (El Bachawati 2015). A waterproofing membrane is the most important layer in the roof structure. It protects a roof, as well as a building and its property, from moisture, rain, snow and other harmful atmospheric effects.

The variety of the offered waterproofing membranes has grown considerably in the last ten years. The standard systems, including the Built-Up-Roof (BUR), are competing with various types of elastic and plastic Single-ply (SP) waterproofing membranes, such as ethylene propylene diene monomer membranes (EPDM), polyvinyl chloride (PVC) membranes, as well as thermoplastic polyolefin (TPO) and modified bitumen membranes (MBM) (Baskaran *et al.* 2007).

Only the membranes made of thick reinforced bitumen polymer materials and synthetic membranes are used for multifunctional roof covering. The roll roofing usually has an additional layer. All materials used should be resistant to rotting (Zavadskas *et al.* 2008).

Gajauskas (2004) provided the following classification of the main types of waterproofing membranes based on the material and the attachment technique used: roll roofing, polyethylene film roofing and mastic roofing. According to the method of attaching waterproofing membranes to the roof surface, they are divided into the following types:

– the membranes that are glued on with mastic;
– the mechanically attached membranes;
– the sprayed-on or painted membranes;
– the membranes that are soldered on.

Elastomeric properties and service life of waterproofing membranes depend not only on the characteristics of the materials, but also on their installation and quality of the equipment used. Now, cold, liquid, spread over or self-adhesive elastomeric products, which differ in chemical composition and the methods of their application, are available on the market. The problems associated with damp-proof roofing may be caused by the changes in atmospheric conditions and decrease in the quality of the performed works (roof base preparation, in particular) (Ustinovichius *et al.* 2012).

Based on the analysis of roofing market presented in TPM (2016), the systems of the designed and repaired roofs distributed as follows according to their popularity. Bituminous roofing is the fastest growing material, consisting of various types of modified bitumen, like SBS (styrene-butadiene-styrene), APP (atactic polypropylene), Modbit (Modified Bitumen), and at the moment covers about 47% of the market. Tile systems make about 26% of the market. Metal roofing makes about 9% of the market. Finally, other roofing systems make about 18% of the market. However, distribution of these materials can vary in concrete countries. Therefore, different architectural trends in concrete countries are stimulating demand, for example, for metal roofs in other markets, where the market share for metal roofing is low.

The main advantages of new roof systems are high installation quality, industrial production and the ensured production control, as well as ready-to-use waterproofing membranes and their systems. Using these products requires more flexible design and application of more rigorous standards (Shohet *et al.* 2004).

Aamont *et al.* (1976) investigated the durability and service life of multifunctional inverted flat roofs. In this study, two types of violations have been determined. The defects observed at the early stage of roof service include premature defects and the defects caused by wearing. The defects of the first type are usually caused by design errors, as well as low quality of the construction works and materials. They can be observed as early as in the first seven years of roof service. The most common design errors include inappropriate drainage system formation, faulty waterway design and unsuitable material selection. A list of errors made in the construction work is, actually, endless, which can be often explained by the inevitably bad weather conditions. However, under normal conditions of production and adequate control, low quality materials are rare. The possibility of occurrence of premature defects in the damp course of multifunctional inverted flat roofs due to thermal stresses has been completely eliminated. Mechanical injuries are also rare, which can be attributed to the fact that a roof is covered by thermal insulating and surface layers. Moreover, a waterproofing layer is not acted upon by the local concentrated operating load.

Based on the tests of durability and chemical stability of the considered roof systems conducted by Aamont *et al.* (1976), the conclusion can be made that multifunctional inverted flat roofs are more advantageous than multifunctional traditional (ordinary) flat roofs in this respect. However, the final conclusion can be only made, when all individual cases are studied.

The external surface (the protective shell) of building enclosures is subjected to the harmful effects of the climatic conditions. Therefore, atmospheric effect is the main factor causing the failure of the roof covering. Atmospheric effects are divided into natural phenomena (rain, snow, wind, sunrays, etc.) and complex chemical and biological processes causing the air pollution (Norvaišienė *et al.* 2003).

To systematize the reports and technical inspection documents, Walter *et al.* (2005) made a classification of flat roof defects. According to the authors, this is an effective tool, helping to avoid different descriptions of the same defects, depending on the subjective opinions of specialists, or using the ambiguous terms.

Cash (1997) conducted the tests on relative durability of the most popular flat roof systems (based on the data for 1996). The reaction to temperature variation was different for various materials. We hope that the results obtained in the discussed work will be useful for designing more durable roof systems.

2. Using MADM methods in construction

Various methods are available for supporting complex decisions in construction. Some of them are as follows: Kaklauskas *et al.* (2012) proposed a Passive House model for quantitative and qualitative analyses; in Kuzman *et al.* (2013) AHP (Analytic Hierarchy Process) was used to compare passive house construction types; Brown *et al.* (2013) used decision making for assessing renovation; in the work of Bucoń and Sobotka (2015), the alternatives of residential building repair are evaluated, while Cannemi *et al.* (2014), Bhanot and Jha (2012) used decision making as a support tool for policy making; in Kalibatas *et al.* (2011, 2012), Zavadskas *et al.* (2016) the indoor environment was evaluated from the perspective of an ideal alternative; in the work of Chen *et al.* (2014) MADM was used for evaluating the impacts of VOC emissions in the US single-family houses; Hopfe *et al.* (2013) assessed building performance; in Książek *et al.* (2015) decision-making in construction project development was described; in Zagorskas *et al.* (2014) MADM was used for evaluating thermal insulation alternatives of historical brick buildings in the Baltic Sea Region and Augutis *et al.* (2014) used MADM for assessing energy infrastructure. The papers of Zavadskas *et al.* (2014), Mardani *et al.* (2015a, 2015b) presented the surveys of MADM methods.

The common features of all MADM methods (Triantaphyllou 2000), as well as MCDM techniques, are based on using the alternatives, representing the available options, and attributes (or criteria) in decision making. Since the attributes or criteria chosen for evaluation often have different dimensions, it is proposed to normalize them, i.e. to translate attributes with different dimensions into the dimensionless attributes. The problem of choosing a MADM method is presented in Chakraborty (2011), Roy and Słowinski (2013). Moreover, there are different ways of developing MADM methods.

In this research, MADM and game-theory methods were used to evaluate different waterproofing layers. For this evaluation, different MADM methods can be used. For simplicity, we chose the SAW method, as well as three game-theory approaches (Hurwicz, Laplace, and Bayes). In the first step, this scheme of evaluation allowed us to perform the objective evaluation of the alternatives. The approaches applied to the analysis and the main formulas used for calculations are given below.

2.1. Complex MADM and game theory-based methods used for evaluating the alternatives

The main steps of the considered method application are given in Figure 1.

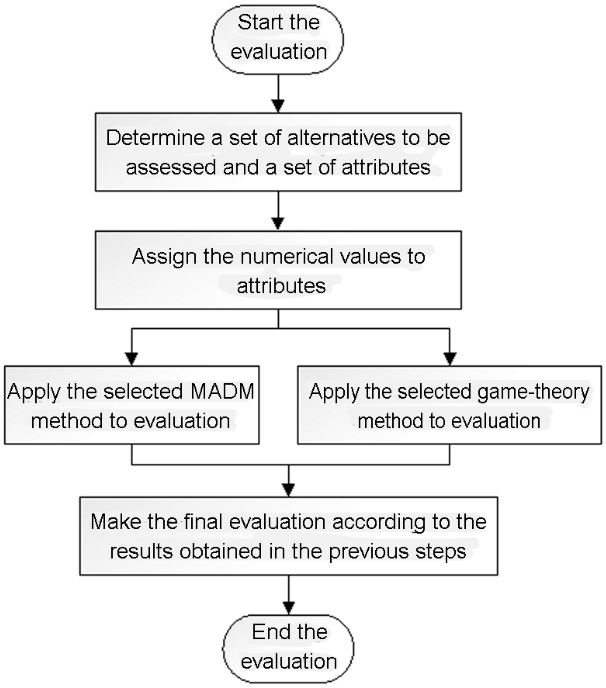

Fig. 1. Complex MADM and game theory-based methods used for evaluating the alternatives

In the step, including the selected MADM method application, we used SAW (MacCrimon 1968; Zavadskas, Kaklauskas 1996), while in the step based on the game theory application, we used three (Hurwicz, Laplace, Bayes) rules, which are described below. According to the application domain, those methods are useful in this analysis to make a decision which alternative is better.

2.2. The SAW method

SAW (Simple Additive Weighting) is based on the sum of the products of the weights (MacCrimon 1968; Zavadskas, Kaklauskas 1996). To determine the effectiveness of an alternative, a decision matrix is formed and normalized. Then, the normalized decision matrix elements are multiplied by the criteria weights and the obtained products are summed up. The sum of products of the most effective alternative will be maximum:

$$A = \{A_i \mid \max_i \sum_{j=1}^{n} q_j \overline{x_{ij}}\}, \quad (1)$$

where: A_i is an alternative, q_j denotes the criteria weights; x_{ij} denotes the elements of the normalized decision matrix: $i = (1, m)$, $j = (1, n)$.

When the SAW method is applied, the elements of a decision matrix are normalized, using linear normalization formulas (MacCrimon 1968). In the analysis, the matrix of the initial data, with the alternatives of waterproofing membranes given in its rows in this case, was constructed first, while the relevant attributes were presented in its columns. Each attribute was maximized or minimized. Then, the attributes were normalized (converted), using maximization and minimization formulas:

$$x_{ij} = q_j \frac{a_j}{a_j^{\max}}; \quad (2) \qquad x_{ij} = q_j \frac{a_j^{\min}}{a_j}, \quad (3)$$

where q_j denotes the weights (significances) of attributes; x_{ij} denotes normalized decision matrix elements: $i = (1, m)$, $j = (1, n)$.

In the next step, a matrix of normalized values was constructed, and each attribute (criterion) was assigned the weight coefficient in such a way that the sum of all coefficients was equal to one.

2.3. Analysis of the alternatives based on using the game theory method

The analysis of the alternatives was made by using three game theory rules (offered by Hurwicz, Laplace, and Bayes) and LEVI 3.0 program. The algorithm of the analysis begins with filling the table of the initial data. Then, weight coefficients, "Significance Factors" are introduced, and a decision matrix is normalized, using the "Transformation" function, while using the "Solution methods" function helps to obtain the matrix of the final data based on the selected methods (Zavadskas *et al.* 2002).

2.3.1. The Hurwicz rule

An optimal strategy is based on the best and the worst results (Hurwicz 1951). These values, calculated from the row's minimum and row's maximum, are integrated into a weighted average using optimism parameters:

$$S_1^* = \left\{ S_{1_i} \middle| S_{1_i} \in S_1 \cap \left\{ S_{1_{i_0}} \middle| h_{i_0} = \max_i h_i ; \right.\right.$$
$$\left.\left. h_i = (1-\lambda); \min_j a_{ij} + \lambda \max_j a_{ij} ; 0 \le \lambda \le 1 \right\}\right\}. \quad (4)$$

The value λ = 1 gives the most pessimistic solution (Hurwicz's rule). For the value of λ = 0 only the maximum (the greatest risk) values are considered.

2.3.2. The Laplace rule

According to this rule, the result was calculated based on the condition that the possibilities of the strategies of all the competitions are equal (Zavadskas *et al.* 2004). The row with the maximal sum was selected:

$$S_1^* = \left\{ S_{1_i} \middle| S_{1_i} \in S_1 \cap \left\{ S_{1_{i_0}} \middle| \frac{1}{n} \sum_{j=1}^{n} a_{i_0 j_0} = \max_i \left(\frac{1}{n} \sum_{j=1}^{n} a_j \right) \right\}\right\}. \quad (5)$$

2.3.3. The Bayes rule

In the literature, this criterion is known as the mean criterion, or Bayes-Laplace principle (Zavadskas *et al.* 2004). The optimal strategy was determined as follows:

$$S_1^* = \left\{ S_{1_i} \middle| S_{1_i} \in S_1 \cap \left\{ S_{1_{i_0}} \middle| \sum_{j=1}^{n} q_j a_{i_0 j} = \max_i \left(\sum_{j=1}^{n} q_j a_{ij} \right); \right.\right.$$
$$\left.\left. \sum_{j=1}^{n} q_j = 1; q_j \ge 0; j = 1, \ldots, n \right\}\right\}. \quad (6)$$

3. Selection of the effective membrane alternative

In this section, the evaluation of the materials available on the market for installing waterproof membranes in multifunctional inverted flat roofs is presented. To solve this problem, ten various alternatives of the types of roofing, i.e. popular types of waterproof roof covering offered on the market, were chosen, taking into account the materials and technological processes used in their production and installation of the described roof type. Then, for the most accurate evaluation of the alternatives, a set of the evaluation criteria was defined based on the available data and specifications.

3.1. Description of the alternatives

The analysis of waterproofing membranes used in the multifunction inverted roof constructions allowed us to choose ten alternatives of these elements. A description of each type of membranes and the material of which they are made, as well as, their brands, are given below and in Table 1:

- A_1 is APP bitumen membrane;
- A_2 is SBS polymer-modified bitumen membrane;
- A_3 is a Self-adhesive membrane made of polymer film (300-HDPE), self-adhesive bitumen rubber layer;
- A_4 is a Synthetic PVC membrane;
- A_5 is TPO thermoplastic synthetic membrane;
- A_6 is TPO/FPO synthetic elastic polyethylene membrane;
- A_7 is a Liquid two-component water-impervious PU membrane;
- A_8 is EPDM synthetic rubber membrane;
- A_9 is EVA/EBA synthetic (ethynyl vinyl acetate/ethynyl ethyl acrylate) membrane;
- A_{10} is PMMA liquid two-component water-impervious material.

3.2. Description of the attributes

In solving the problem of selecting the effective membranes for considered roofs, the quantitative and qualitative criteria were classified based on the analysis of the characteristics of the products and the materials they were made of, which were provided by the manufacturers, as well as taking into account economic calculations, analytical surveys and other data. The most relevant eight criteria, describing the effectiveness of the considered alternatives were found to be as follows. Values for criteria were taken from the cost estimation in construction system Sistela[1], where all costs are based on Lithuanian market, or values presented by manufacturers.

- x_1 is the cost of one square meter of the material (€/m²), a general economic factor, consisting of two components. The first component is the cost given in euros by manufactures in the official price-list. The second component is the cost of the damp course (membrane) installation (€/m²), according to the prices from Sistela in 2016-03;
- x_2 is workability, a general factor expressed in points from 1 to 10, showing feasibility of the membranes' installation characterized by labour expenditure, the need for using various mechanisms and tools in the

[1] http://www.sistela.lt/samatu-skaiciavimas

Table 1. The initial data of decision matrix (A_i alternatives, X_i decision attribute)

Criteria		The cost of one sq. m of the material	Workability	Longitudinal tensile strength	Breaking elongation	Vapour diffusion resistance	Nail tear resistance	Flexibility at negative temperature	Maximal guarantee period
Units of measure		€/m^2	points	N/50 mm	%	μ	N	°C	years
Weight of the criteria ($\sum q = 1$)		0.25	0.2	0.05	0.05	0.15	0.05	0.1	0.15
Opt. direction		min	max	max	max	max	max	max	max
Alternatives	A_1	18.19	6	200	350	20000	400	15	20
	A_2	19.23	7	700	45	20000	220	15	20
	A_3	21.84	8	215	324	90000	125	15	20
	A_4	26.02	1	550	200	20000	150	25	30
	A_5	22.02	3	1200	150	150000	150	40	30
	A_6	16.86	2	450	550	150000	150	35	20
	A_7	30.97	10	350	450	2000	33	5	20
	A_8	19.88	4	300	300	58000	30	45	30
	A_9	35.55	5	650	40	25000	150	20	20
	A_{10}	27.04	9	800	35	2050	400	5	25

process, etc. It is a qualitative maximized criterion used for comparing the membranes, which is expressed in points from Sistela in 2016;

- x_3 is longitudinal tensile strength (N/50 mm), also referred to as tensile strength, showing the highest tensile strength of the kind, which the material can withstand prior to failure;
- x_4 is breaking elongation (%), the material property, reflecting its relative elongation or capability of elongating up to the limit of failing. It is a qualitative maximized criterion expressed in percent;
- x_5 is vapour diffusion resistance (μ-value), the ratio of water vapour permeability of the air to water vapour permeability of the considered material, showing how many times the latter is larger than the permeability coefficient of the air layer of the same thickness. It is a maximized qualitative attribute;
- x_6 is nail tear resistance (N), showing the membrane's tear resistance. It is a quantitative maximized attribute expressed in newtons;
- x_7 is flexibility at negative temperature (°C), showing the property of the material to be elastic and flexible at a negative temperature given on the Celsius scale. It is a maximized criterion;
- x_8 is a maximal guarantee period, given in years, which is declared by the manufacturer. This guarantee is assured, if a waterproofing membrane is installed following all the rules and recommendations provided, as well as using the materials and products recommended by the manufacturer.

3.3. Weights of attributes

The weights of attributes q_j were determined empirically, using questioning of experts (e.g. homeowners' associations and construction companies). In Table 1 weights are presented in raw 3. Weights (q_j) of attributes was calculated using Eqn (7) as follows:

$$q_j = \frac{s_j}{\sum_{j=1}^{m} s_j}, \tag{7}$$

where s_j denotes the significance of the j-th attribute, j denotes number of attributes. In our research, we have 8 attributes. Therefore, $m = 8$ and significance s_j can be an integer number in a range [1; 8], where 8 is the most preferable attribute and 1 is the least preferable attribute. Moreover, significance of all attributes for the same alternative should be different. The correctness of experts' answers was determined as in Kendall (1970). The result is acceptable.

3.4. Analysis of the alternatives by using the SAW method

First, a matrix of the initial data (Table 1), with the membrane alternatives for roofs given in its rows and the relevant attributes provided in its columns, is constructed. The criteria are normalized (converted), using Eqns (2) and (3).

The final data matrix is given in Table 2. After summing up the final criterion values assigned to each alternative (Sum (A)), they are presented in the descending order.

The results of the analysis of the alternatives by using the SAW method based on the final matrix data (Table 2) allowed the authors to arrange the alternatives in the priority order as follows: $A_5 > A_6 > A_8 > A_3 > A_1 > A_{10} > A_2 > A_7 > A_9 > A_4$. Based on the obtained data, the most effective alternative out of the ten analysed options was found to be a TPO thermoplastic synthetic membrane (A_5).

Table 2. The final decision matrix data

Criteria		x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	Sum (A)	Rank
Alternatives	A_1	0.232	0.12	0.008	0.032	0.02	0.05	0.033	0.1	0.595	5
	A_2	0.219	0.14	0.029	0.004	0.02	0.028	0.033	0.1	0.553	7
	A_3	0.193	0.16	0.009	0.03	0.09	0.016	0.033	0.1	0.630	4
	A_4	0.162	0.02	0.023	0.018	0.02	0.019	0.056	0.15	0.487	10
	A_5	0.191	0.06	0.05	0.014	0.15	0.019	0.089	0.15	0.743	1
	A_6	0.25	0.04	0.019	0.05	0.15	0.019	0.078	0.1	0.705	2
	A_7	0.136	0.2	0.015	0.041	0.002	0.004	0.011	0.1	0.509	8
	A_8	0.212	0.08	0.013	0.027	0.058	0.004	0.1	0.15	0.644	3
	A_9	0.119	0.1	0.027	0.004	0.025	0.019	0.044	0.1	0.498	9
	A_{10}	0.156	0.18	0.033	0.003	0.002	0.05	0.011	0.125	0.581	6

Table 3. The decision matrix of the final data obtained following the Hurwicz rule

Criteria		x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	Result	Rank
Weight of the criteria ($\sum q = 1$)		0.25	0.2	0.05	0.05	0.15	0.05	0.1	0.15		
Alternatives	A_1	18.19	6	200	350	20000	400	15	20	0.116	2
	A_2	19.23	7	700	45	20000	220	15	20	0.109	3
	A_3	21.84	8	215	324	90000	125	15	20	0.092	7
	A_4	26.02	1	550	200	20000	150	25	30	0.075	9
	A_5	22.02	3	1200	150	150000	150	40	30	0.105	4
	A_6	16.86	2	450	550	150000	150	35	20	0.125	1
	A_7	30.97	10	350	450	2000	33	5	20	0.096	6
	A_8	19.88	4	300	300	58000	30	45	30	0.100	5
	A_9	35.55	5	650	40	25000	150	20	20	0.044	10
	A_{10}	27.04	9	800	35	2050	400	5	25	0.089	8

3.5. Analysis of the alternatives in terms of the game theory approach

The evaluation of the alternatives based on using Hurwicz, Laplace and Bayes rules and LEVI 3.0 program (Zavadskas *et al.* 2002) is described in this section.

3.5.1. Evaluation based on the Hurwicz rule

The evaluation by Eqn (4), where the value $\lambda = 0.5$ gives the most realistic solution, allowed us to obtain the decision matrix of the final data (Table 3).

As shown in Table 3, the best alternative is A_6, presenting a TPO/FPO synthetic elastic polyethylene membrane.

3.5.2. Evaluation based on the Laplace rule

The evaluation according to the Laplace rule by Eqn (5) yielded the decision matrix of the final data (Table 4).

As shown in Table 4, the best alternative is A_5, presenting a TPO synthetic membrane.

3.5.3. Evaluation based on the Bayes rule

The evaluation according to the Bayes rule by Eqn (6) yielded the decision matrix of the final data (Table 5).

As shown in Table 5, the best alternative is A_5, presenting a TPO synthetic membrane.

4. Main results and discussion

We start our discussion from the analysis of the chosen criteria and their weights. As was explained in Section 3.3, weights of attributes were determined by experts, who based on their practice assign significance values to the criteria. From Table 1 it can be seen that the cost (x_1; 0.25) is the most significant criterion, when choosing an appropriate membrane. At the second place according to the weights, we have workability (x_2; 0.2) and vapour diffusion resistance (x_5; 0.15) with maximal guarantee period (x_8; 0.15). The less significant are flexibility at negative temperature (x_7; 0.1) and longitudinal tensile strength (x_3; 0.05), breaking elongation (x_4; 0.05) and nail tear resistance (x_6; 0.05). As can be concluded here, economical factors and water resistance, expressed by criteria x_5, are the most significant criteria when choosing a membrane. This factor can be explained as follows. Economic factors and guarantee are significant for all people first of all, since the final price depends on the chosen alternative. The significance of a water resistance factor depends

Table 4. The decision matrix of the final data obtained following the Laplace rule

Criteria		x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	Result	Rank
Weight of the criteria ($\sum q = 1$)		0.25	0.2	0.05	0.05	0.15	0.05	0.1	0.15		
Alternatives	A_1	18.19	6	200	350	20000	400	15	20	0.058	5
	A_2	19.23	7	700	45	20000	220	15	20	0.055	7
	A_3	21.84	8	215	324	90000	125	15	20	0.062	4
	A_4	26.02	1	550	200	20000	150	25	30	0.049	8
	A_5	22.02	3	1200	150	150000	150	40	30	0.086	1
	A_6	16.86	2	450	550	150000	150	35	20	0.072	3
	A_7	30.97	10	350	450	2000	33	5	20	0.039	9
	A_8	19.88	4	300	300	58000	30	45	30	0.077	2
	A_9	35.55	5	650	40	25000	150	20	20	0.024	10
	A_{10}	27.04	9	800	35	2050	400	5	25	0.056	6

Table 5. The decision matrix of the final data obtained following the Laplace rule

Criteria		x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	Result	Rank
Weight of the criteria ($\sum q = 1$)		0.25	0.2	0.05	0.05	0.15	0.05	0.1	0.15		
Alternatives	A_1	18.19	6	200	350	20000	400	15	20	0.090	5
	A_2	19.23	7	700	45	20000	220	15	20	0.089	6
	A_3	21.84	8	215	324	90000	125	15	20	0.095	4
	A_4	26.02	1	550	200	20000	150	25	30	0.065	8
	A_5	22.02	3	1200	150	150000	150	40	30	0.112	1
	A_6	16.86	2	450	550	150000	150	35	20	0.101	3
	A_7	30.97	10	350	450	2000	33	5	20	0.058	9
	A_8	19.88	4	300	300	58000	30	45	30	0.108	2
	A_9	35.55	5	650	40	25000	150	20	20	0.027	10
	A_{10}	27.04	9	800	35	2050	400	5	25	0.079	7

on the climatic zone of Lithuania, since we have a lot of precipitation as rain and snow. After this discussion, we can forward to the results of our survey.

Based on the analysis of the alternatives by the SAW method and in terms of the game theory (following Hurwicz, Laplace and Bayes rules), which is also included in the group of multi-attribute decision making methods, a summary table of the data yielded by all the methods involved in the analysis was derived (Table 6).

As can be seen in Table 6, the SAW method and the approaches based on the game theory, following the Hurwicz, Laplace and Bayes rules, may be effectively used for evaluating the considered criteria from various perspectives to determine the best alternative of multi-functional inverted roof membranes. The solution of the considered problem by using all the discussed methods yielded the same result, showing that the alternative A_5 was the best. This was the TPO thermoplastic synthetic membrane.

Determining the criteria weights (significances) by using the SAW method, the authors could observe that, in the case, when the criteria values did not differ considerably, the final result was not too sensitive. The alternative A_5 demonstrated a relatively low cost and good operational characteristic. Therefore, it was considered to be the most suitable alternative of the impervious membrane

Table 6. The summary table of the data obtained in the analysis performed by all methods

The methods used	The priority order of the alternatives	The final priority order
The SAW method	$A_5 > A_6 > A_8 > A_3 > A_1 > A_{10} > A_2 > A_7 > A_9 > A_4$	$A_5 > A_6 > A_8 > A_1 > A_3 > A_2 > A_{10} > A_7 > A_4 > A_9$
The Hurwicz rule	$A_6 > A_1 > A_2 > A_5 > A_8 > A_7 > A_3 > A_{10} > A_4 > A_9$	
The Laplace rule	$A_5 > A_8 > A_6 > A_3 > A_1 > A_{10} > A_2 > A_4 > A_7 > A_9$	
The Bayes rule	$A_5 > A_8 > A_6 > A_3 > A_1 > A_2 > A_{10} > A_4 > A_7 > A_9$	

and was recommended for installing in the considered type of roofs. Following our discussion, the main reason why alternative A_5 wins this assessment is that, according to the discussion of weights of criteria, presented at the beginning of this section, this alternative (A_5) has the best values for criteria, which are the most significant (see Table 1).

Conclusions

The analysis of multifunctional inverted flat roofs performed in the work helped the authors to determine the criteria required for evaluating various types of multifunctional inverted flat roof covering. The attributes selected included the cost of the square meter of roofing (€/m^2); workability (points); longitudinal tensile strength ((N/50 mm); breaking elongation (%); vapour diffusion resistance, μ-value; nail tear resistance (N); flexibility at negative temperature (°C) and maximal guarantee period (years). Based on the selected attributes, ten popular types of waterproofing roof covering available on the market have been evaluated.

Based on the analysis of the multicriteria decision-making method used in construction, the method for solving the problem considered in the present work was offered. It was a complex MADM and game theory-based method. This approach does not depend on any particular MADM method, and therefore, can be widely used and adapted to any case. For more detailed and accurate evaluation, SAW and three game theory methods were also used in this work.

The evaluation performed by using the described method allowed the authors to conclude that the alternative A_5, representing the TPO thermoplastic synthetic membrane, was the best of all available options of waterproof roofing because it was not expensive and had optimal technical characteristics.

The performed work demonstrated the need for further research and analysis in the considered area, with the aim of finding the ways of increasing the reliability of multifunctional flat roofs. It can be recommended to introduce multi-criteria evaluation into the practice of all design organisations as a powerful auxiliary tool, which can ensure effective decision-making and higher building quality.

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