NUMERICAL SIMULATION OF RAILWAY TRACK SUPPORTING SYSTEM USING FINITE-INFINITE AND THIN LAYER ELEMENTS UNDER IMPULSIVE LOADS

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Abstract. The present study deals with two dimensional, numerical simulation of railway track supporting system subjected to dynamic excitation force. Under plane strain condition, the coupled finite-infinite elements to represent the near and far field stress distribution and thin layer interface element was employed to model the interfacial behavior between sleepers and ballast. To account for the relative debonding, slipping and crushing that could take place in the contact area between the sleepers and ballast, modified Mohr-Coulomb criterion was adopted. Furthermore an attempt has been made to consider the elasto-plastic material non-linearity of the railway track supporting media by employing different constitutive models to represent steel, concrete and supporting materials. Based on the proposed physical and constitutive modeling a code has been developed for dynamic loads. The applicability of the developed F.E code has been demonstrated by analyzing a real railway supporting structure.

Keywords: elasto-plastic, dynamic loading, finite-infinite elements, interface elements modeling, railway, track supporting media.

1. Introduction

In railway transport, there is an ongoing demand for performance increase, which is driven by the need to keep a competitive edge against other modes of transport, such as aircrafts, cars and ships. This requires advance technical requirements in their analysis and design procedures. Most of the railway track system consists of rails, sleepers, ballast, sub-ballast and sub-grades. It is essential to use numerical model that realistically represents the actual behavior of this track system subjected to actual load. Most of the investigators make use of finite element for the purpose of physical and material modeling. For example, a linear elastic analysis was performed for rail track support systems by Desai *et al.* (1982), using a three dimensional finite isoparametric solid elements for the discretization purpose. To simulate static linear dynamic response of the moving train on track supporting media, Kok (1997) has used the following elements:

 − Thimoshenko beam to model the railway track supporting media;
 − Four-node element to represent the sleeper;
 − Gap element to account for the contact between the sleepers and ballast.

The response of the railway-track-supporting structure has been discussed with respect to displacements, bending moments and shear forces. The authors ignored effect of flexibility of the bedding system.

Lombaert *et al.* (2006) proposed a 2.5-dimensional model which account for rail, rail pad, floating slab and slab mat by ignoring the track soil interface. Steenbergen and Metrikine (2007) used the classical model that is beam on an elastic half space to model slab-track railway system subjected to vertical axle of a running train. The study was focused on the effect of the interface modeling between the beam and half space on the dynamic response of the track surrounding soil. The finite element modeling was employed to develop a dynamic model incorporating concrete sleeper and ballast. The emphasis was placed on partial and full interaction between the sleeper and ballast (Kaewunruen, Remennikov 2007).

The response of supported structure which can be described using a 2.5-D finite element model, subjected to a moving or stationary harmonic loading was formulated by Sheng *et al.* (2005).

A general numerical model was developed by Galvı́n *et al.* (2010a) to analyze the High-Speed-Train (HST) dynamic interactions and their effects on nearby structures. The model was analyzed in 3-D using finite element and boundary element formulations. The study was done for two cases, in first case; the train speed was lower than the Rayleigh wave velocity in the soil, while

in the second case it was higher. In both cases the computed results were in a good agreement with experimental results.

Another numerical study by means of a 2.5D coupled finite element boundary element model was carried out by Galvı́n *et al.* (2010b) in order to prediction of railway induced vibrations. One case is considered a ballasted track on an embankment using two alternatives models. In the first model the ballast and embankment were modeled by 2.5 solid elements in continuum mechanics, while the second model was simplicity represented. The comparison of the results in both methods with the real measurements at a site in Reugng, France, showed there is a very big difference due to disregarding in the simplified representation.

From literatures, it can be concluded that most of the researchers employed numerical or semi numerical methods to simulate the railway track-sleeper-ballast-sub-ballast-supporting soil system. There is no or little literature available on complete finite element modeling railway-track-ballast-sub-ballast and supporting soil under dynamic loading. There is no literature on effect of soil nonlinearity on overall responses of railway track supporting system. The present study is continuation of authors' previous work (Noorzaei *et al.* 2009) where the nonlinear response of railway track and supporting structures were investigated under static loads. This study focuses on the following objectives:

a) To develop a numerical tool using finite element technique which is able to integrate the railway track supporting media as a single compatible unit when the system subjected to dynamic loadings such as impact loads;

b) To consider elasto-plastic constitutive law for the materials involved in railway track supporting media;

c) To develop a F.E. code based on the items a and b;

d) To evaluate the safety of railway track supporting system under dynamic loads.

However, in the present study the effect of track structure has been neglected.

2. Proposed physical modeling of the railway track supporting system

The following elements are used to represent the railway track supporting system:

− The eight-node isoparametric element is used to model the railway track- sleepers and supporting media (Zienkiewicz 1983).

− Five-node infinite element to represent the far field behavior. The coupling of this element with conventional finite element was presented by Viladkar *et al.* (1991) and Noorzaei *et al.* (1994).

− Six node thin layer element to account for the interfacial behavior between the sleepers and ballast.

These elements along with their functions are published in authors' previous article (Noorzaei *et al.* 2009).

3. Constitutive modeling

In the problem of railway Track-Support system the following yield criterions were adopted.

3.1. Von Mises yield criterion

The Von Mises yield criterion suggested that yielding occurs when $\left(J_2'\right)$ reaches a critical value t:

$$\left(J_2'\right)^{1/2} = Q(k),\tag{1}$$

where: (k) = a material parameter to be determined; $\left(J_2'\right)$ = the second deviatory stress variant.

The Von Mises yield criterion was utilized to represent the elasto-plastic behavior of the railway track.

3.2. Drucker-Prager yield criterion

The Drucker-Prager yield criterion (Drucker, Prager 1952) is expressed as:

$$\alpha J_1 + \left(J_2'\right)^{1/2} = k,\tag{2}$$

where:

$$\alpha = \frac{2\sin\varphi}{\sqrt{(3)(3-\sin\varphi)}};\tag{3a}$$

$$k' = \frac{6c\cos\varphi}{\sqrt{(3)(3-\sin\varphi)}};\tag{3b}$$

(J_1) = the first stress invariant of deviatoric stress components; φ = friction angle; c = coefficient value of material.

The elasto-plastic behavior of the sleeper, ballast and sub-ballast was represented by *Drucker-Prager* yield criterion.

3.3. Mohr-Coulomb yield criterion

The Mohr-Coulomb yield criterion is expressed as:

− Elastic constitutive relationship.

The interface behaviour between the sleeper and ballast in this analysis was represented by 6-node thin layer interface having thickness t (Noorzaei *et al.* 2009). The elastic global constitutive matrix $[D_e]$ is presented by:

$$[D_e] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & tk_n & 0 \\ 0 & 0 & tk_s \end{bmatrix},\tag{4}$$

where: $[D_e]$ = elasticity matrix for thin layer element; tk_n, tk_s = the shear and normal stiffness respectively; t = thickness of the thin layer.

Within the elastic range the behavior of the thin layer element can be described by the conventional elastic relations:

$$\{d\varepsilon\}^e = \begin{bmatrix} D \end{bmatrix}_e \{d\sigma\} ; \qquad (5)$$

− Elasto–plastic constitutive relationship.

Theory of plasticity has been used, particularly to describe the failure, yield or ultimate behaviour of interface, while the pre-failure behaviour is assumed to be linear elastic. During an increment of stress $d\sigma$, the incremental stress-strain relationship for an isotropic elastic-plastic material can be expressed in terms of elastic and plastic strain parts. Thus:

$$\{d\varepsilon\} = \{d\varepsilon^e\} + \{d\varepsilon^p\} . \qquad (6)$$

In elasto-plastic region time effects are considered and the corresponding strain vector is calculated based on:

$$\{d\sigma\} = [D]_{ep}\{d\varepsilon\} , \qquad (7)$$

where:

$$D_{ep} = \left[[D] - \frac{[D]\{a\}\{a\}^T [D]^T}{A + \{a\}^T [D]\{a\}} \right] , \qquad (8)$$

where: f = yield function; Q = potential function; $\begin{bmatrix} D_{ep} \end{bmatrix}$ = elasticity matrix; A = hardening parameter.

To simulate the relative slipping, debonding and crushing that could take place in the contact area between the rail and ballast, modified Mohr-Coulomb criterion was adopted. Fig. 1 illustrates these regions. An elasto-plastic constitutive law $\begin{bmatrix} D_{ep} \end{bmatrix}$ was evolved in explicit form for different yielding regions (Noorzaei *et al.* 2009).

3.3.1. Region I: Debonding mode of deformation

The elasto-plastic matrix $\begin{bmatrix} D_{ep} \end{bmatrix}$ = for this case is:

$$\begin{bmatrix} D_{ep} \end{bmatrix} = \frac{t}{k_s\tau^2 f_t^4 + k_n\sigma_n^2 c^4}$$
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & k_s k_n \tau^2 f_t^4 & -k_s k_n \tau\sigma_n c^2 f_t^2 \\ 0 & -k_s k_n \tau\sigma_n c^2 f_t^2 & k_s k_n \sigma_n^2 c^4 \end{bmatrix} . \qquad (9)$$

3.3.2. Region II: Slip mode of deformation

The elasto-plastic matrix $\begin{bmatrix} D_{ep} \end{bmatrix}$ = for this case is:

$$\begin{bmatrix} D_{ep} \end{bmatrix} = \frac{t}{k_s + k_n\gamma} \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_n k_s & k_n k_s \tan\varphi \\ 0 & k_n k_s \tan\varphi & k_s k_n \gamma \end{bmatrix} . \qquad (10)$$

3.3.3. Region III: Crushing mode of deformation

The elasto-plastic matrix $\begin{bmatrix} D_{ep} \end{bmatrix}$ for this case is:

$$\begin{bmatrix} D_{ep} \end{bmatrix} = \frac{t}{k_s\tau^2 f_c^4 + k_n\sigma_n^2 c^4}$$
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & k_s k_n \tau^2 f_c^4 & -k_s k_n \tau\sigma_n c^2 f_c^2 \\ 0 & -k_s k_n \tau\sigma_n c^2 f_c^2 & k_s k_n \sigma_n^2 c^4 \end{bmatrix} , \qquad (11)$$

where: σ_n, τ = normal and shear stress respectively; f_t, f_c = tensile and compressive strength respectively; k_s, k_n = the shear and normal stiffness respectively; t = thickness; φ = friction angle; $\gamma = \tan\phi.\tan\varphi$; ψ = dilatancy angle; C = cohesion.
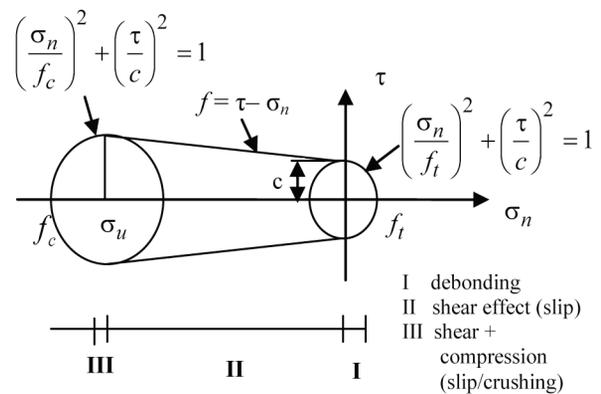

Fig. 1. Modified Mohr-Coulomb criterion (Linsbauer, Bhattacharjee 1999)

4. Time marching computational Scheme for dynamic analysis

The equation of motion for an inelastic system obtained from the consideration of equilibrium of forces is given by:

$$[M]\{\ddot{x}\} + \{q\}(x, \dot{x}) = \{f\} , \qquad (12)$$

where: $[M]$ is the mass matrix of the system; $\{\ddot{x}\}$ is the acceleration vector, is the vector of internal resisting force which depends upon the displacement $\{x\}$ and velocity $\{\dot{x}\}$ and $\{f\}$ is the externally applied load vector.

The internal resisting forces are defined by the stiffness matrix $[k]$ and damping matrix $[C]$.

The Newmark's (1959) predictor-corrector (Owen, Hinton 1980) has been adopted for the dynamic solution. In the Newmark's predictor-corrector, the following relations are defined:

$$[M]\{\ddot{x}\}_{t+\Delta t} + \{q\}(x_{t+\Delta t}, \dot{x}_{t+\Delta t}) = \{f\}_{t+\Delta t} , \qquad (13)$$

where:

$$\{x\}_{t+\Delta t} = \{\bar{x}\}_{t+\Delta t} + \beta(\Delta t)^2 \{\ddot{x}\}_{t+\Delta t} ; \qquad (14a)$$

$$\{\dot{x}\}_{t+\Delta t} = \{\bar{\dot{x}}\}_{t+\Delta t} + \Delta t\gamma\{\ddot{x}\}_{t+\Delta t} ; \qquad (14b)$$

$$\{\overline{x}\}_{t+\Delta t} = \{x\}_t + \Delta t\{\dot{x}\}_t + 0.5(\Delta t)^2(1-2\beta)\{\ddot{x}\}_t \; ; \; (14c)$$

$$\{\overline{\dot{x}}\}_{t+\Delta t} = \{\dot{x}\}_t + \Delta t(-\gamma)\{\ddot{x}\}_t . \tag{14d}$$

Here β and γ are the parameter that control the stability and accuracy of the method. The quantities $\{\overline{x}\}_{t+\Delta t}$, $\{\overline{\dot{x}}\}_{t+\Delta t}$ are historical values and $\{x\}_{t+\Delta t}$, are the corrector values.

In the inelastic solution for thin layer element, the stiffness matrix and damping matrix are reformulated to take into account the effect of cracking, crushing, yielding, opening and slipping that may occur at the interface between the railway track and sleeper.

The Newmark's algorithm for each time step is applied as follows (Newmark 1959):

(1) Set iteration counter $j = 0$;

(2) Predict the response in term of displacement, velocity and acceleration corresponding to values at time t_{n+1}:

$$\{x\}_{t+\Delta t}^j = \{\overline{x}\}_{t+\Delta t} = \{x\}_t + \Delta t\{\dot{x}\}_t + 0.5(\Delta t)^2(1-2\beta)\{\ddot{x}\}_t \; ; \; (15a)$$

$$\{\dot{x}\}_{t+\Delta t}^j = \{\overline{\dot{x}}\}_{t+\Delta t} = \{\dot{x}\}_t + \Delta t(1-\gamma)\{\ddot{x}\}_t \; ; \tag{15b}$$

$$\{\ddot{x}\}_{t+\Delta t}^j = \left[\{x\}_{t+\Delta t}^j - \{\overline{x}\}_{t+\Delta t}\right]\frac{1}{\beta\Delta t^2} \; ; \tag{15c}$$

(3) Evaluate residual forces using the following Equation:

$$\{r\}^j = \{f\}_{t+\Delta t} - [M]\{\ddot{x}\}_{t+\Delta t}^j - [C]\{\dot{x}\}_{t+\Delta t}^j - [K]\{x\}_{t+\Delta t}^j .(16)$$

The matrix $[K]$ is modified for each element if cracking, crushing, yielding, opening and slipping within element occurs;

(4) Generate effective stiffness matrix using the following relation:

$$[K]^* = [M]\frac{1}{\beta\Delta t^2} + [C]^j\frac{\gamma}{\beta\Delta t} - [K]^j \; ; \tag{17}$$

(5) Solve for incremental displacement:

$$[K]^*\{\Delta x\}^j = \{r\}^j \; ; \tag{18}$$

(6) Update displacement, acceleration and velocity vectors:

$$\{\Delta x\}_{t+\Delta t}^{j+1} = \{x\}_{t+\Delta t}^j + \{\Delta x\}^j \; ; \tag{19a}$$

$$\{\ddot{x}\}_{t+\Delta t}^{j+1} = -\left[\{x\}_{t+\Delta t}^{j+1} - \{\overline{x}\}_{t+\Delta t}\right]\frac{1}{\beta\Delta t^2} \; ; \tag{19b}$$

$$\{\dot{x}\}_{t+\Delta t}^{j+1} = \{\dot{x}\}_{t+\Delta t} + \gamma\Delta t\{\ddot{x}\}_{t+\Delta t}^{j+1} \; ; \tag{19c}$$

(7) If $\{\Delta x\}^j$ or $\{r\}^j$ do not satisfy the convergence conditions, then set $j = j+1$ and go to step (3), otherwise continue the next step;

(8) If convergence is achieved; set:

$$\{x\}_{t+\Delta t} = \{x\}_{t+\Delta t}^{j+1} \; ; \tag{20a}$$

$$\{\dot{x}\}_{t+\Delta t} = \{\dot{x}\}_{t+\Delta t}^{j+1} \; ; \tag{20b}$$

$$\{\ddot{x}\}_{t+\Delta t} = \{\ddot{x}\}_{t+\Delta t}^{j+1} \tag{20c}$$

for use in the next time step. Also set $t = t + \Delta t$ to begin the next step.

5. Finite element computation program

The proposed finite element idealization and constitutive modeling for different material involved in the problem of railway track-sleeper-ballast-sub-ballast and soil mass have been implemented into the existing two-dimensional finite element (Noorzaei *et al.* 2005) elasto-plastic dynamic finite element program extensively modified in view of inclusion of the thin layer element and including the following features:

- – Multi-elements;
- – Static linear analysis and elasto-plastic analysis under static loading;
- – Multi-yield criterion elasto-plastic analysis;
- – Elasto- plastic analysis under dynamic loads.

6. Analysis of railway track bedding system subjected to dynamic loads

In Malaysia, a typical 5-meter high embankment of double tracking system is usually adopted. The UIC 54 kg rails (169 mm) are provided over the concrete sleepers, which are placed at 600 mm spacing. The sleepers rest over a 300 mm (minimum) thick ballast layer and 300 mm thick sub-ballast layer. The typical geometry of the railway track bedding system is shown in Fig. 2 (Noorzaei *et al.* 2009). The nonlinearity of the soil has been taken into consideration due to nonlinear nature of the soil using Mohr-Coulomb's Elasto-Plastic model. Material properties are shown in Table 1. Based on earlier works (Desai *et al.* 1986), the material properties used for thin layer element are:

$$k_{nn} = 100\,000 \text{ kN/cm}^2, \; k_{ss} = 100\,000 \text{ kN/cm}^2,$$
$$\varphi = 30° \text{ and } c = 0.7.$$

The railway track supporting system was idealized under plane strain condition (Esveld *et al.* 1996; Noorzaei *et al.* 2009). Fig. 3 shows the finite model for railway track supporting media through finite, infinite and thin-layer elements. The thickness of ballast and sub-ballast are 300 mm each. The total number of nodes, elements and types of element used in the finite modeling are also shown in Fig. 3. Fig. 4 shows the impact load with duration applied here is 0.015 sec for the Malaysian Railway System. The dynamic response of the railway supporting structure with respect to displacements, accelerations, principal stresses and yielding pattern are presented in the following discussion. Time domain dynamic analysis was carried out to study the behavior of the railway-track-solid media.
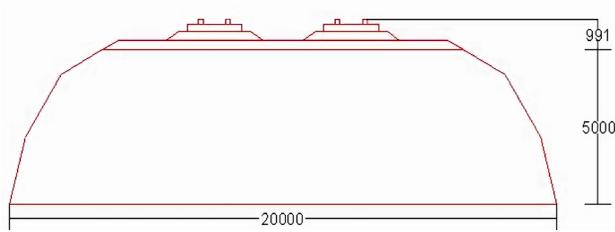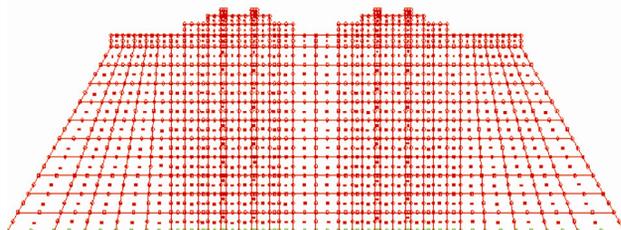
Fig. 2. Typical geometry for railway track

Type of element	No. of elements
8-Node finite element	1846
6-Node infinite element	64
6-Node thin-layer element	12

(Total of nodes = 5854)

(Total of Element = 1922, included thin layer elements)

Fig. 3. Finite-infinite-thin layer idealization of railway track sleeper-ballast-sub-ballast-supporting media
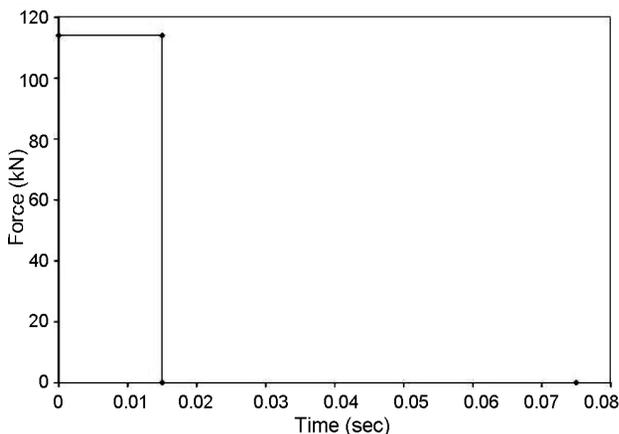

Fig. 4. Impulse-shape load (KTMB 2003)

Table 1. Material properties

No.	Node No.	Maximum vertical acceleration (mm/sec^2)
1	1211	1.4×10^5
2	1141	1.22×10^4
3	1065	1.09×10^3
4	711	7.5×10
5	5707	2.75×10^{-4}

7. Results and discussion

In order to present the time history of the railway-track supporting media with respect to displacements and accelerations, five nodes are selected. Locations of these 5 nodes are shown in Fig. 5.
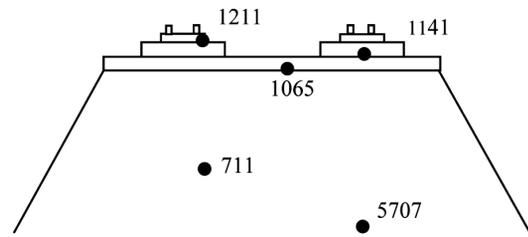

Fig. 5. Location of five nodes selected

The maximum vertical displacement of these 5 nodes has been shown in Table 2. As it was expected the nodes situated near to the zone of the impact load display higher displacements than the nodes located far away from the point of load application. The horizontal displacement is too small compared to the vertical displacement.

Table 2. Variation of maximum vertical displacement for selected nodes

No.	Node No.	Maximum vertical displacement (mm)
1	1211	2.2
2	1141	0.146
3	1065	0.00425
4	711	0.000195
5	5707	1.95×10^{-8}

Table 3 presents the maximum variation of acceleration in vertical direction for selected nodal points as shown in Fig. 5. Since the dynamic load is applied in vertical direction as shown in Fig. 4, the horizontal acceleration is negligible. Also from this table it can be observed that far nodes from the acting load display the smaller acceleration.

Table 3. Variation of maximum vertical acceleration for selected nodes

| Material Properties				
Item	Dynamic elastic modulus (MPa)	Poisson ratio	Density (KN/m^3)	Friction angle
Rails	246 000	0.2	78.5	
Sleeper	36 000	0.2	24.5	
Ballast	72 000	0.3	15	30
Sub-ballast	60 000	0.3	15	25
Sub-grade 1	30	0.3	12	20
Sub-grade 2	32	0.3	12	20
Sub-grade 3	36	0.3	12	15
Sub-grade 4	42	0.3	11	15
Sub-grade 5	42	0.3	11	10
Sub-grade 6	54	0.3	10	10
Sub-grade 7	66	0.3	10	10

Figs 6a and b show the variation of horizontal and vertical displacements along depth of railway track-sleeper-ballast-soil media respectively. From these plots, it is clear that deformation at the top of the railway track supporting system was evaluated and tends marginal

value at the bottom. It is obvious from these plots that evaluated deformations are maximum at the top and reducing along the depth.

7.1. Displacements
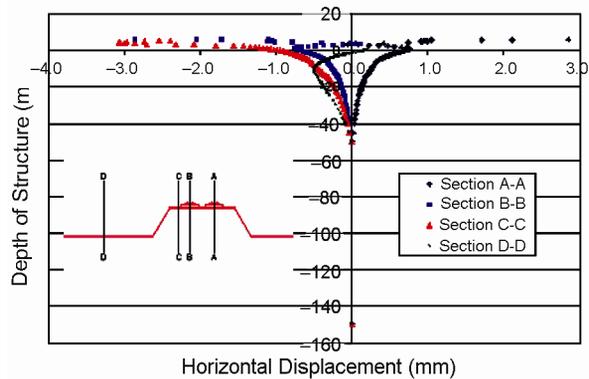
a) Horizontal displacement
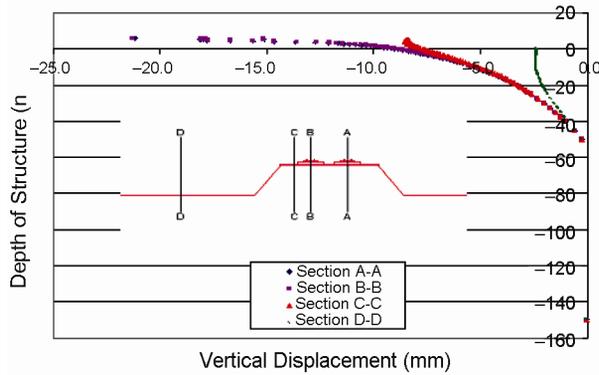
b) Vertical displacement

Fig. 6. Variation of displacements
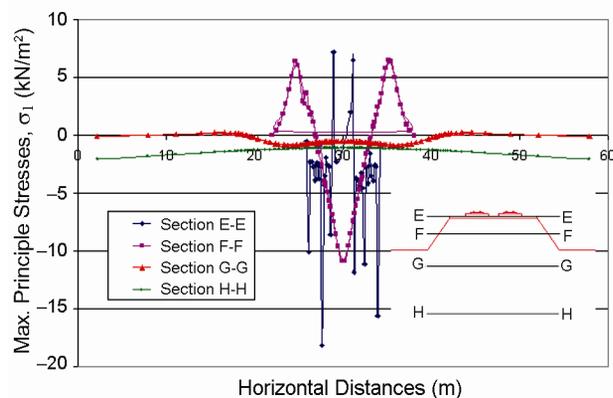
7.2. Stresses

Fig. 7. Distribution of maximum principal stresses (σ_1)

Figs 7 and 8 show the distribution of maximum principal stress (σ_1) and minimum principal stress (σ_3) along horizontal planes of railway track supporting system respectively with various sections namely Section E-E, F-F, G-G and H-H. It is clear from the plots that stresses are high (–54, –18 and +7.5 kN/m²) at the points
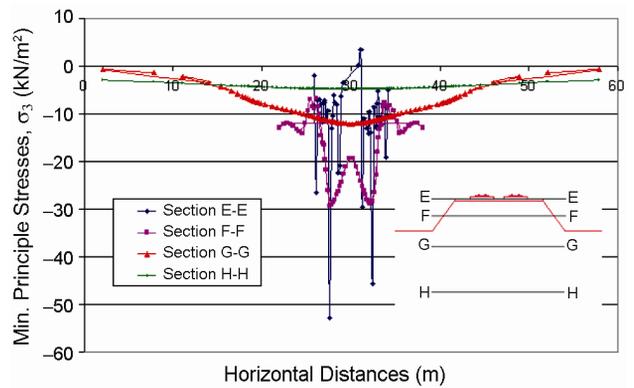

Fig. 8. Distribution of minimum principal stresses (σ_3)

of load impact and could lead to failure. Hence there is a need to do safety evaluation.

8. Yielding pattern and safety evaluation of the system

In order to assess the safety of the railway supporting structures an attempt has been made to carry out the following parametric studies.

− Under constant Δt the load magnitude in Fig. 10 was varied to 5, 10 and 25 times of the marginal intensity;
− The duration of impulsive was taken as $\Delta t = 0.15$, 0.20 and 0.5 sec respectively.

Fig. 9 illustrates the spread of plastic flow in the railway supporting structures for L.F = 5, 10 and 25 respectively. It can be noticed from this plot that the plots of plastic flow indicate that railway supporting structure can stand impulse load with load factor = 25 and with minimum number of yield Gauss points.
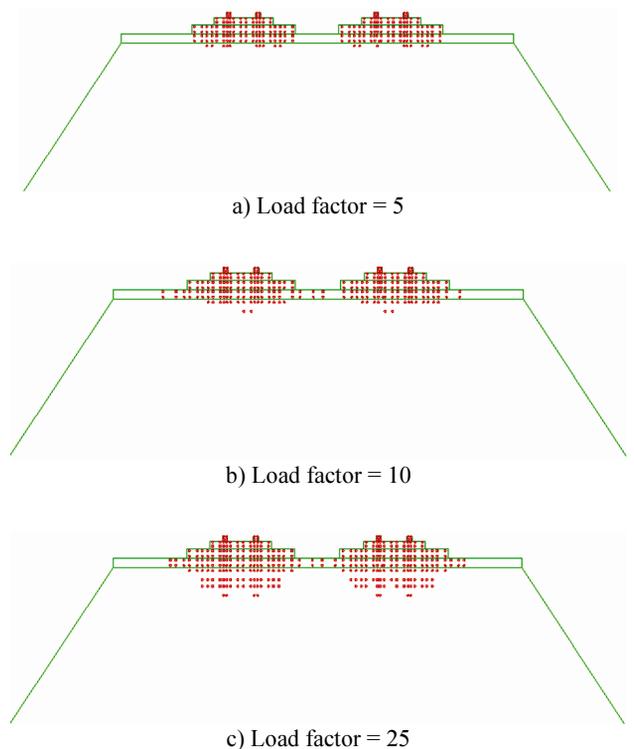

a) Load factor = 5

b) Load factor = 10

c) Load factor = 25

Fig. 9. Yielding behavior with different load factor

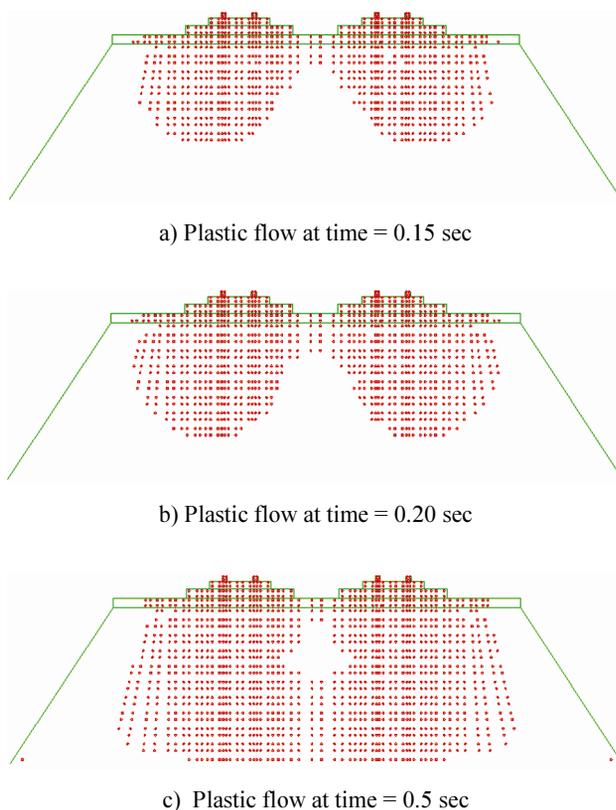Figs 10a–10c show the yielding behavior with duration of time equal 0.15, 0.20 and 0.50 sec respectively.

a) Plastic flow at time = 0.15 sec

b) Plastic flow at time = 0.20 sec

c) Plastic flow at time = 0.5 sec

Fig. 10. Spread of plastic flow at different duration time

From the results, it can be summarized that yielding initially starts from the rail and sleeper and then flows vertically downwards and finally toward the centre of railway line.

9. Conclusions

The primary purpose of the present study is to develop a 2-D dynamic finite element code with multi-element nature. The conventional finite-infinite-thin layer was employed to represent the railway track supporting media. Multi yield criterion concept was used to represent the stress-strain relationship of different materials involved in railway supporting structures. The validity of the developed program code was established against simple examples. Furthermore based on the particular railway track-supporting media analyzed the following primary specific conclusion can be drawn:

- The maximum displacement evaluated for impulsive dynamic load was 22 mm;
- There was no significant spread of plastic flow under varying dynamic load magnitude;
- Special attention should be paid when the duration of impulsive load is increased from 0.15 to 0.5 sec, since there is completed plastic flow in the region of railway track- supporting media;
- Yielding modes starts from the rail and then flow vertically to the centre of railway line implement-

ed of thin-layer element able to capture worst yielding behavior;
- The prediction of failure modes in railway track–supporting system is very important information for future maintenance works. From result of elasto-plastic analysis, the safety factor recommended is between 3 and 5.

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GELEŽINKELIO BĖGIŲ SISTEMOS SKAITMENINIS MODELIAVIMAS NAUDOJANT BAIGTINIUS BEGALINIUS IR PLONASIENIUS ELEMENTUS, VEIKIAMUS DINAMINĖMIS APKROVOMIS

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S a n t r a u k a

Šiame straipsnyje nagrinėjama geležinkelio bėgių dvimatė sistema, veikiama dinaminės žadinančios jėgos. Pagal plokštuminę deformacijų sąlygą suporuoti baigtiniai begaliniai elementai aprašo artimą ir tolimą įtempių pasiskirstymo sritis, o plonasienis sąsajos elementas yra įvestas sąveikai tarp pabėgių ir balasto aprašyti. Yra naudojamas modifikuotas Mohr-Coulombo kriterijus, kuris įvertina atplėšimo, nuslydimo ar išspaudimo galimybes kontaktiniame pabėgių ir balasto paviršiuje. Taip pat yra pabandyta įvertinti tampriai plastinės medžiagos netiesiškumą, sudarant skirtingus sudėtinius sistemos modelius (pavyzdžiui, iš plieno, betono, atramų medžiagų). Remiantis pasiūlytu fiziniu ir sudėtiniu modeliavimu sudarytas skaičiavimo algoritmas dinaminėms apkrovoms įvertinti. Sudaryto baigtinių elementų algoritmo pritaikymas yra pademonstruotas išnagrinėjus realią geležinkelio bėgių sistemą.

Reikšminiai žodžiai: tampriai plastinė medžiaga, dinaminė apkrova, baigtiniai begaliniai elementai, sąsajos elementų modeliavimas, geležinkelio bėgiai, bėgių kelio atramos.

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