

A MODEL FOR STRENGTH AND STRAIN ANALYSIS OF STEEL FIBER REINFORCED CONCRETE

Gediminas Marčiukaitis¹, Remigijus Šalna², Bronius Jonaitis³, Juozas Valivonis⁴

*Department of Reinforced Concrete and Masonry Structures, Vilnius Gediminas Technical University,
Saulėtekio al. 11, LT-10223 Vilnius, Lithuania*

E-mails: ¹gelz@vgtu.lt; ²Remigijus.Salna@vgtu.lt (corresponding author);

³Bronius.Jonaitis@vgtu.lt; ⁴Juozas.Valivonis@vgtu.lt

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Abstract. The article proposes a model for strength and strain analysis of steel fiber reinforced concrete (SFRC). The model is based on general principles for creating and modelling structural composites and on reinforced concrete code. Differently from other examples, the elastic and plastic properties of the components (concrete and steel) of the introduced model are directly taken into account. The model gives an opportunity to determine tension and compression strength, the elasticity modulus of fiber concrete and the main parameters of its elasticity and plasticity. A good agreement between the obtained results and those of experiments performed by other investigators was confirmed. Differences between the ratios of theoretical and experimental values are insignificant and vary within the limits of 1.06–1.10. This model may be used for the analysis of reinforced concrete members reinforced by steel fibers (SFRC) in a dispersible way assuming stress distribution diagrams.

Keywords: composite, concrete, steel fiber, fiber reinforced concrete, strength, secant modulus, elastic and plastic strain.

1. Introduction

Concrete reinforced with steel fibers is a composite material the properties of which differ from those of concrete and steel fibers when taken separately. Concrete properties are mainly changed by steel fibers: strength and strain at tension, flexion, and elasticity modulus are increased and other mechanical properties are enhanced.

The areas of using fiber concrete may be both non-structural and structural. Fibers enable to control plastic strain and moisture movements of concrete, and consequently the process of cracking. In structural sense, fibers can be substituted for complicated reinforcement with bars and in the case of a combined stress state, enable to avoid sudden failure due to an action of static and dynamic loads etc.

The application of SFRC for various stiff joints of reinforced concrete structures was well known long ago as an effective choice for additional reinforcement in the case of a combined stress state due to its distribution to various chaotic directions (Li 2002; Šalna and Marčiukaitis 2007; Szmigiera 2007; Özcan *et al.* 2009; Chalioris and Karayannis 2009; Brandt 2008). However, the application of steel fibers to load bearing structures has been strictly limited for a long time due to a lack of the regulated methods of analysis.

These factors encouraged various countries (USA, Japan, Russia etc.) to work out documents regulating the use of steel fibers in the form of additional supplements to design codes. Furthermore, it is implicitly stressed in

modern research work that the application of steel fibers to stiff joints, such as a connection between a column and a slab, is expedient. In such case, not only the strength of the structure is increased but also failure becomes predictive – the brittle failure mode is superseded by the plastic one. The application of such concrete is expedient for pavements on bridges, airports, tunnels and the like because of roughness, resistance to cracking and abrasion of their surface (Li 2002; Johnston and Zemp 1991; Meddah and Bencheikh 2009; Maleki and Mahoutian 2009; Kasper *et al.* 2008; Chiaia *et al.* 2009).

Steel fibers provide a possibility of producing and applying thin slabs with various shapes of the surface in accordance with architectural solutions to buildings.

The extensive use of fiber concrete for load bearing structures is confined by different existing methods for determining properties and especially by the absence of any method defining the properties of such concrete in the elastic-plastic state of stress.

It should be emphasized that when reinforcing concrete with fibers under compression, plastic deformations originate at the stress of $0.4\sigma_{cu}$. Concrete ultimate stress σ_{cu} corresponds to stress in steel fibers only of $(0.1...0.3)\sigma_y$. In such case, for relationship $(\sigma-\varepsilon)$, there will be limits within which both components will deform linearly; however, above the limit, one component deforms elastically while the other – plastically (Fig. 1). According to the classical theory of composites, limit stress in the composite is determined by criterion:

$$dF / d\varepsilon = 0, \quad (1)$$

where F – force acting the composite.

Then, according to the additive law, we obtain:

$$\frac{dF}{d\varepsilon} = \left(\frac{dF_f}{d\varepsilon} \right) V_f + \left(\frac{dF_c}{d\varepsilon} \right) (1 - V_f). \quad (2)$$

This condition can be accurately solved when complete diagrams of σ – ε for components are known. Nevertheless, in the majority of cases, the linear relationship of ($\varepsilon_i = \sigma_i / E_i$) is assumed and the utilization of component properties is evaluated using empirical coefficients. Diagrams in Fig. 1 indicate that their accurate description is complicated. For common concrete, as suggests EC 2, σ – ε diagram may be approximated by broken lines, i.e. σ – ε diagram is resolved into a geometrical regular parabolic, triangular, trapezium and even rectangular diagrams or into a combination of those. For usual reinforced concrete, three shapes are used: 1) parabola and rectangular, 2) triangular and rectangular and 3) rectangular only. Our investigations showed that the second form was the simplest and the closest one to the real diagram. Moreover, considering fiber concrete, some theoretical attempts to assume the simplified diagram were made. Maalej and Li (1994), Kanda *et al.* (2000), Soranakom and Mobasher (2008), Bareišis and Kleiza (2004) proposed employing idealized σ – ε diagrams for SFRC. Various methods of examining SFRC were created (Pupurs *et al.* 2006; Li and Wang 2002; Nelson *et al.* 2002; Li 1992; Wang *et al.* 1989; Zhang and Li 2002; Kanda and Li 1999; Kanda *et al.* 2000; Leung and Li 1991; Maalej *et al.* 1995; Stang *et al.* 1995; Волков *et al.* 2007; Kang *et al.* 2010; Olivito and Zuccarello 2010, Fantilli *et al.* 2009) the analysis of which shows that three groups of models can be distinguished:

1. strength of SFRC is determined on the basis of the additive law;

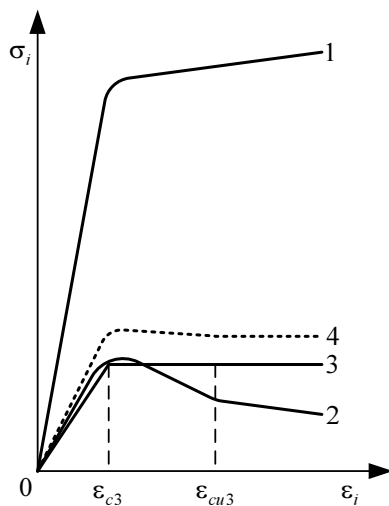


Fig. 1. Schematic σ – ε diagrams: for steel fiber (1), actual for concrete (2), for concrete according to EC 2 (3), for steel fiber reinforced concrete (4)

2. strength of SFRC is determined using the principles of the mechanics of failure;
3. strength of SFRC is determined using empirical relationships.

The main principle of the majority examined methods for the analyses of SFRC is the additive law (Малмейстер *et al.* 1980); however, they differ in the assumptions of determining correction coefficients. Some authors reduce a chaotic distribution of steel fibers to the regularly orientated one (Филлинс and Харис 1980; Рабинович 2004; Marčiukaitis 1998), some of those create models according to probabilistic principles (Wang and Becker 1989) while others determine experimentally or do not reduce chaotic distribution to regular one but introduce empirical coefficients (Li 1992; Stang *et al.* 1995; Kanda and Li 1999; Kanda *et al.* 2000; Zhang and Li 2002).

The principles of determining the strength of fiber concrete using the principles of failure mechanics were analyzed by Li (1992), Maalej *et al.* (1995), Kanda *et al.* (2000), Zhang and Li (2002), Zhang *et al.* (2001). An agreement with experimental results is good but the theoretical apparatus is complicated and practically almost was not used.

The methods of analyses based on empirical data belong to the third group (Harajli *et al.* 1995; Narayanan and Darwish 1987). Though this method is frequently applied, still, the actual performance of the composite components is almost not evaluated. The analysis of investigation results presented by the above mentioned as well as by other authors shows that the most precise results are obtained using the additive law. Great differences in relation to experimental results are obtained because plastic deformations in tension and compression concrete have not been taken into account.

2. The Proposed Model for Strength and Strain Analysis of SFRC

A comparison and analysis of models for strength analysis of SFRC have showed that there is no united opinion how strength and strains are to be determined. In the methods proposed by the majority of authors and in some standards for designing composites, using either σ – ε composite diagram determined experimentally (ASTM C1018; JSCE-SF4; NBP No.7; NFP 18-409; TR-34) or employing general principals of designing composites using additional (mostly empirical) service coefficients for components is recommended.

The suggested model for strength and strain analysis of SFRC is based on general principles of creating and modelling composites (additive law) with a direct evaluation of elastic and plastic characteristics of composite materials.

Following classical assumptions of creating and designing composites one can write:
the strength of the composite in the general case is

$$\sigma_{sfrc} = \sigma_f V_f \psi_f + \sigma_c (1 - V_f), \quad (3)$$

elasticity modulus of the composite in the general case is

$$E_{sfrc} = E_f V_f \psi_f + E_c (1 - V_f), \quad (4)$$

where ψ_f – the coefficient of service for materials depending on their joint action, strain properties, the quantity and orientation of inclusions, anchorage properties etc. It can be mostly determined by modelling; at a later stage, correction is made conducting experiments.

Formulas 3 and 4 show that for describing composite stress, to strain relationships expressed via variation in the elasticity modulus of materials is required.

For describing stress to strain relationships with materials, the following assumptions were applied:

1. for steel inclusions – linear σ - ε relationship (since elastic steel strains are much less in comparison with ultimate concrete strains);
2. for concrete – trapezoidal – the expression of elastic strains is $\varepsilon_{c,el} = f_c / E_c$, plastic ones – $\varepsilon_{c,pl} = f_c / E_c (1 - \lambda_c)$ and ultimate strains – $\varepsilon_{c,u} = f_c / E_c (1 - \lambda_{cu})$.

In this case, plasticity coefficients are as follows:

$$\begin{cases} \lambda_c = \frac{\varepsilon_c E_c - f_c}{\varepsilon_c E_c}, \\ \lambda_{cu} = \frac{\varepsilon_{cu} E_c - f_c}{\varepsilon_{cu} E_c}. \end{cases} \quad (5)$$

The performed tests revealed that the plasticity coefficient of concrete can be defined in the following way:

$$\lambda_c = 1 - 0.061 f_c^{0.5}. \quad (6)$$

3. For composite – trapezoidal, strains are expressed as follows: elastic $\varepsilon_{sfrc,el} = f_{sfrc} / E_{sfrc}$, plastic – $\varepsilon_{sfrc,pl} = f_{sfrc} / E_{sfrc} (1 - \lambda_{sfrc})$, ultimate – $\varepsilon_{sfrc,u} = f_{sfrc} / E_{sfrc} (1 - \lambda_{sfrc,u})$.

Under load composite, inclusion and matrix deform together, and therefore it can be expressed as

$$\varepsilon_{sfrc} = \varepsilon_c = \varepsilon_f. \quad (7)$$

The relation between strains and stress at the elastic stage when $\varepsilon_c \leq f_c / E_c$ is

$$\begin{cases} \sigma_c = \varepsilon_c E_c, \\ \sigma_f = \varepsilon_f E_f. \end{cases} \quad (8)$$

Eqs 7 and 8 show that the strain of the composite equals to:

$$\varepsilon_{sfrc} = \sigma_c / E_c = \sigma_f / E_f. \quad (9)$$

Using the law of mixtures and formula (9), composite stress equals to

$$\begin{aligned} \sigma_{sfrc} &= \frac{\sigma_f}{E_f} (E_c V_c + E_f V_f) = \\ \sigma_f &\left(\frac{E_c}{E_f} V_c + V_f \right) = \sigma_f (\alpha_f V_c + V_f), \end{aligned} \quad (10)$$

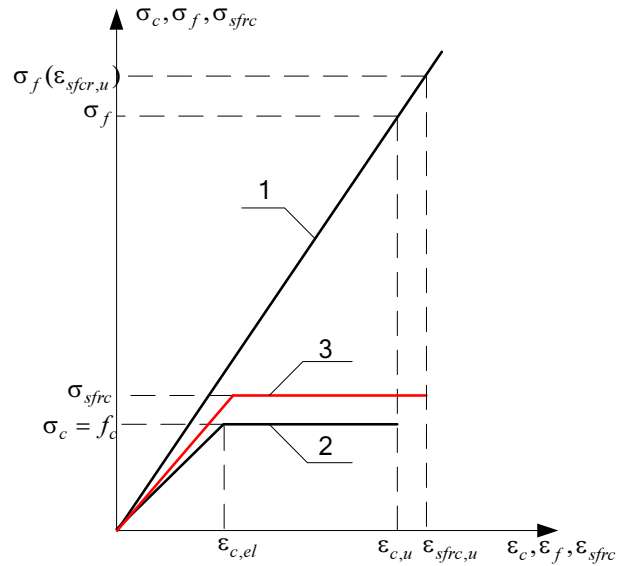


Fig. 2. Idealized σ - ε diagrams: 1 – steel inclusions; 2 – concrete; 3 – the composite

where $\alpha_f = E_c / E_f$ when $\varepsilon_{sfrc} \leq f_c / E_c$; and

$\alpha_f = E_c (1 - \lambda_c) / E_f$ when $f_c / E_c < \varepsilon_{sfrc} \leq \varepsilon_{c,u}$.

Formulas (9) and (10) show that the strength of the composite depends on two main parameters – the value of inclusion stress σ_f and composite strain ε_{sfrc} , while ultimate composite strength – on ultimate inclusion stress $\sigma_{f,u}$ and ultimate composite strain $\varepsilon_{sfrc,u}$. Formula (10) is valid when $\varepsilon_{sfrc} \leq \varepsilon_{c,u}$. However, ultimate composite strain $\varepsilon_{sfrc,u}$, when inclusion strength is greater than matrix strength $f_s / f_c \geq 1$, exceeds concrete matrix strain $\varepsilon_{c,u}$ (Nelson *et al.* 2002; Li and Wang 2002; Li 1992; Wang *et al.* 1989; Рабинович 2004; Zhang and Li 2002; Kanda and Li 1999; Kanda *et al.* 2000; Leung and Li 1991; Stang *et al.* 1995) since inclusion can resist acting stress (especially tensile one). Using trapezoidal σ - ε diagram for the composite (Fig. 2), the ultimate strain of composite $\varepsilon_{sfrc,u}$ may be expressed via composite elasticity modulus:

$$\varepsilon_{sfrc,u} = \frac{\sigma_{sfrc}}{E_{sfrc} v_{sfrc,u}} = \frac{\sigma_c}{E_{sfrc} (1 - \lambda_{sfrc,u})}. \quad (11)$$

Elasticity coefficient for composite $v_{sfrc,u}$ can be expressed via the ultimate strains of the concrete matrix assuming that $\varepsilon_{c,u} = \varepsilon_{sfrc}$ and $v_{sfrc,u} = v_{sfrc}$, and using the iteration method, a more accurate value of $v_{sfrc,u}$ can be obtained. According to this assumption and equating strains of the composite and concrete, one can write:

$$\frac{\sigma_{sfrc}}{E_{sfrc} v_{sfrc}} = \frac{\sigma_c}{E_c v_c}. \quad (12)$$

Using the law of mixtures for determining composite stress σ_{sfrc} and elasticity modulus E_{sfrc} and making mathematical rearrangements from relation (12), the coefficient of elasticity for the composite is equal to

$$v_{sfrc} = \frac{\left(V_c + \frac{\sigma_f}{\sigma_c} V_f \right) v_c}{\left(V_c + \frac{E_f}{E_c} V_f \right)}. \quad (13)$$

Relation (13) clearly shows that the coefficients of elasticity and plasticity for the composite along with ultimate strain depend on the ratio of stresses σ_f / σ_c acting in the inclusion and concrete matrix. This ratio is determined using the general principles of work produced by external and internal forces. In the same way, inclusion stress σ_f may be expressed via reduced cross-section of concrete:

$$\sigma_f = \frac{N}{A_{eff}} = \frac{N}{A_c \frac{E_f}{E_c} + A_f} \quad (14)$$

and similarly concrete stress σ_c expressing via reduced cross-section of steel fibers:

$$\sigma_c = \frac{N}{A_{eff}} = \frac{N}{A_c + \frac{E_f}{E_c} A_f}. \quad (15)$$

From (14) and (15), it is obvious that the ratio of σ_f / σ_c equals to:

$$\frac{\sigma_f}{\sigma_c} = \frac{A_c \alpha_f + A_f}{A_c + A_f \alpha_f}, \quad (16)$$

where $\alpha_f = E_f / E_c$.

Since $(A_f + A_c)l = 1$ or $V_f + V_c = 1$, then after mathematical rearrangements, equation (16) can be put in this way:

$$\frac{\sigma_f}{\sigma_c} = \frac{\alpha_f - V_f (\alpha_f - 1)}{1 + V_f (\alpha_f - 1)}. \quad (17)$$

Putting expression (17) into expression (13), the elasticity coefficient of the composite at ultimate strain can be expressed by:

$$\left\{ \begin{array}{l} v_{sfrc} = \frac{\left(1 - V_f + \frac{\alpha_f - V_f (\alpha_f - 1)}{1 + V_f (\alpha_f - 1)} V_f \right) v_c}{(1 - V_f + \alpha_f V_f)} \\ \lambda_{sfrc} = 1 - v_{sfrc} \end{array} \right. \quad (18)$$

The values of elasticity coefficient v_{sfrc} determined by equation (18) and those of ultimate composite strain $\varepsilon_{sfrc,u}$ determined by equation (11) are compared with the experimental ones.

The second parameter from formulas (9) and (10) to be considered is stress σ_f in the inclusion of the composite. Formula (10) indicates that composite strength σ_{sfrc} practically depends on the value of stress σ_f in inclusion. The ultimate stress value in inclusion, i. e. strength f_y of inclusion, can be reached when inclusion is properly anchored (Laranjeira *et al.* 2010; Šalna and Marčiukaitis 2010). According to the classical theory of reinforced concrete, anchorage strength depends on bond stress τ_f between the matrix and inclusion and the area of the bond. Then, in the case of full anchorage, the following condition has to be satisfied:

$$\tau_f d_f l_{f,an} \pi = f_y A_f. \quad (19)$$

From equation (19), required anchorage length $l_{f,an}$ is determined or taking analogous $l_{f,an}$ the ultimate bond stress can be obtained. However, when steel fiber is bent, the bond stress is supplemented with additional tangential stress at the bend. When the bond stress is noted by τ_1 and tangential stress at the bend τ_2 , then the total bond stress τ_f can be written in the form of:

$$\tau_f = \tau_1 + \tau_2. \quad (20)$$

When tangential stress τ_2 is expressed in the form of product $\tau_2 = \tau_1 k_{at}$, then (20) can be presented by:

$$\tau_f = \tau_1 + \tau_2 = \tau_1 + \tau_1 k_{at} = \tau_1 (1 + k_{at}). \quad (21)$$

The expression of the average normal stress σ_f in steel fibers via tangential ones τ_f and the application of formula (22) give expression for determining normal stress according to the geometrical parameters of steel fibers and evaluation of different influence of the bend on the anchorage:

$$\sigma_f = \tau_f \frac{l_f}{d_f} = \tau_1 (1 + k_{at}) \frac{l_f}{d_f}. \quad (22)$$

The coefficient of effectiveness k_{at} for the bend depends on the type of steel fibers, bending shape and failure type at pull-out and has to be determined conducting tests. Coefficient k_{at} values determined performing experiments (Šalna and Marčiukaitis 2010) are presented in Table 1.

Table 1. The average tangential stress at the bend and the values of coefficient k_{at}

Steel fiber type	τ_2 , MPa	k_{at}	Coefficient dispersion k_{at}	Variation in the coefficient k_{at}
MPZ 60	9.13	2.20	0.11	0.05
MPZ 50	10.75	2.59	0.06	0.02
MPS 50	15.60	3.76	0.48	0.13
MPD 50	9.71	2.34	0.51	0.22
MPG 32	10.71	2.58	0.24	0.09

When the stress of steel fibers only (22) is known, it is possible, according to (4), to determine the strength of the composite applying to regularly orientated inclusions. The analysis of the models proposed by the majority of authors and comparison with test results have showed that when reducing chaotic reinforcement by inclusions into the uniaxial one, the most accurate results are obtained using the model suggested by Рабинович (2004) by means of a product of coefficients allowing for the probability of steel fibers to get into design plane (λ_{op}) and coefficient (λ_p) evaluating the orientation of the introduced reinforcement in relation to design plane:

$$\lambda_{op}\lambda_p = 0.41. \quad (23)$$

After comparing formulas (3, 10, 22, 23), the strength of the composite equals to:

$$\sigma_{sfrc} = 0.41\sigma_f \left(\frac{E_c(1-\lambda_c)}{E_f} V_c + V_f \right) = 0.41(1+k_{at})\tau_1 \frac{l_f}{d_f} \left(\frac{E_c(1-\lambda_c)}{E_f} V_c + V_f \right). \quad (24)$$

On the basis of the adequacy of formulas (3, 4, 24), the coefficient allowing conditions for service ψ_f may be expressed via the ultimate stress of steel fibers. Then, formulas (3) and (4) can be rewritten in the forms:

$$\sigma_{sfrc} = 0.41 \frac{\sigma_f}{\sigma_{fu}} \sigma_{fu} V_f + \sigma_c (1 - V_f), \quad (25)$$

$$E_{sfrc} = 0.41 \frac{\sigma_f}{\sigma_{fu}} E_f V_f + E_c (1 - V_f). \quad (26)$$

Thus, in the developed formulas (25) and (26), the strength and elasticity modulus of SFRC are evaluated and depend on the plasticity coefficient value (5). The ratio of the elasticity modulus of steel fibers to that of concrete gives an opportunity to determine composite stress in relation to the value of plastic strain.

3. Experimental Investigation into the Strength and Strain Properties of SFRC and Comparison with the Model

Test specimens made three main series of different strength of SFRC. The first SFRC series was intended for investigating the anchorage of steel fibers in the concrete matrix using steel fibers of type MZP 50. A concrete mix was produced under laboratory conditions. The quantity of steel fibers corresponding to 1, 1.5 and 2 percent of the volume mass was interblended into the concrete mix in the laboratory. The composition of steel fibers and the concrete mix is presented in Table 2.

The compression strength of SFRC was determined testing under the standard of 150×150×150 mm cubes and that of 100×100×400 mm prisms. The tension strength of SFRC was determined by bending 100×100×400 mm prisms. The elasticity (and strain)

moduli of SFRC in the tension and compression processes were determined by means of measuring tension and compression strains of experimental specimens employing electrical resistance strain gauges. Totally, four specimens in each series were tested.

Table 2. A composition of concrete for the experimental program

Material name	Material quantity kg/m ³		
	FRC series	I series	II series
Cement(42,5R)	320	308.67	312.14
Sand (0–4 mm)	773	947.33	933.53
Gravel(4–16 mm)	1180	924.00	912.54
Water	163	124.67	126.9
Plasticizer	–	1.36	1.37
Steel fibers of MZP 50 type	0; 78.5; 117.75; 157	0; 78.5; 117.75; 157	0; 78.5; 117.75; 157

Strains were measured at the geometrical centres of prism sides in transverse and longitudinal directions using glued electric resistance gauges of 50 mm base length (Fig. 3). The load was increased up to failure in the steps of 20 kN and sustained for 5 min at each step. Load increasing speed was 0.05 kN/s. Apparent elastic limit assumed at the load limit equals to $0.4F_u$.

For assessing experimental plasticity, the value of coefficient λ_{sfrc} for SFRC at tension and compression as well as the elasticity modulus of concrete for compression were determined by the compression of standard 100×100×140 mm prisms and that for tension – by bending the above introduced prisms. The elasticity modulus of the specimens subjected to bending was determined by means of measuring strains of the tension layer in the zone of pure bending with two electrical resistance strain gauges and one inductance strain gauge. Electrical resistance strain gauges provide a possibility of measuring the ultimate strain only for concrete prisms subjected to bending. In SFRC prisms, cracks opened and the ultimate strain and crack width were determined with inductance strain gauge only.

When the ultimate strain and ultimate stress are obtained, experimental plasticity coefficients $\lambda_{sfrc,c,obs}$ and $\lambda_{sfrc,t,obs}$ for SFRC at compression and tension are determined according to formula (5). The values of theoretical

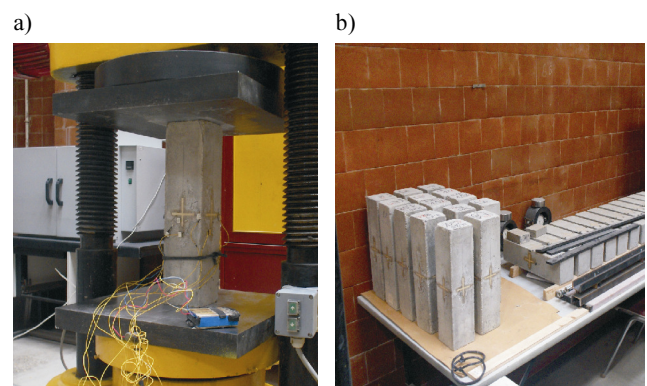


Fig. 3. An experiment on determining the elasticity modulus of SFRC (a); a series of prisms prepared for the test (b)

plasticity coefficient $\lambda_{sfrc,c,cal}$, $\lambda_{sfrc,t,cal}$ are obtained from analysis according to the proposed model. A comparison of experimental and theoretical values is presented in Table 4. The values of theoretical plasticity coefficient $\lambda_{sfrc,c,cal}$, $\lambda_{sfrc,t,cal}$ show a good agreement with the experimental ones. The values of a very small coefficient (0.01 and 0.02, Table 4) of variation demonstrate that the characters of plasticity coefficients both at tension and compression coincide closely.

The average experimental and theoretical values of modulus of deformation, compression and tension strength of SFRC are presented in Table 3, 5, respectively.

A comparison of experimental and theoretical values shows that compression strengths and modulus of deformation of SFRC coincide strongly ($0.98 \div 1.06$). An agreement between theoretical and experimental tension strength values is slightly worse ($0.95 \div 1.18$). After the regression analysis of experimental investigations, the plasticity coefficient of SFRC was determined. It equals to

$$\lambda_{sfrc,c} = \lambda_c \left(1 + 0.008V_f^{1.5}\right). \quad (27)$$

Table 3. A comparison of the values of experimental and theoretical elasticity modulus of SFRC

Series	V_f , %	$E_{c,obs}$, GPa	$E_{sfrc,obs}$, GPa	$E_{sfrc,cal}$, GPa	$\frac{E_{sfrc,obs}}{E_{sfrc,cal}}$
C	0	37.3	–	–	–
FRC1	1.0	–	38.2	37.38	1.02
FRC1,5	1.5	–	39.3	37.42	1.05
FRC2	2.0	–	39.8	37.46	1.06
I	0	32.95	–	–	–
	1.0	–	33.76	33.07	1.02
	1.5	–	34.67	33.14	1.05
II	0	35.50	–	–	–
	2.0	–	35.93	35.7	1.05

Table 4. A comparison of the values of experimental and theoretical plasticity coefficient of SFRC

Series	V_f , %	$\lambda_{sfrc,c,obs}$	$\lambda_{sfrc,t,obs}$	$\lambda_{sfrc,c,cal}$	$\lambda_{sfrc,t,cal}$	$\frac{\lambda_{sfrc,c,obs}}{\lambda_{sfrc,c,cal}}$	$\frac{\lambda_{sfrc,t,obs}}{\lambda_{sfrc,t,cal}}$
C	0	0.706	0.61	–	–	–	–
FRC1	1.0	0.71	0.981	0.649	0.8952	1.09	1.10
FRC1,5	1.5	0.717	0.982	0.6503	0.8955	1.10	1.10
FRC2	2.0	0.721	0.983	0.6517	0.8959	1.11	1.10
I	0	0.694	0.562	–	–	–	–
	1.0	0.702	0.98	0.6655	0.9048	1.05	1.08
	1.5	0.73	0.982	0.668	0.9052	1.09	1.08
II	0	0.712	0.58	–	–	–	–
	2.0	0.719	0.984	0.652	0.9016	1.10	1.09
Average						1.09	1.09
Square deviation						0.02	0.01
Coefficient of variation						0.02	0.01

Table 5. The average values of experimental and theoretical compression and tension strength regarding SFRC

Series	$f_{sfrc,cube,obs}$, MPa	$f_{sfrc,t,obs}$, MPa	$f_{sfrc,cube,cal}$, MPa	$f_{sfrc,t,cal}$, MPa	$\frac{f_{sfrc,cube,obs}}{f_{sfrc,cube,cal}}$	$\frac{f_{sfrc,t,obs}}{f_{sfrc,t,cal}}$
C	50.88	5.71	–	–	–	–
FRC1	53.78	6.78	52.81	5.73	1.02	1.18
FRC1,5	54.03	8.21	53.77	6.93	1.00	1.18
FRC2	56.37	8.95	54.73	8.13	1.03	1.10
I	37.94	4.41	–	–	–	–
	40.10	5.1	40	4.86	1.00	1.05
	40.29	5.75	41.02	6.07	0.98	0.95
II	41.56	5.13	–	–	–	–
	46.04	7.3	45.6	7.49	1.01	0.97

Empirical expression (27) clearly points out that variation in the plasticity coefficient of SFRC and the quantity of steel fibers is not significant and mostly depends on the plasticity coefficient of concrete itself. It agrees with the proposed model (18). According to (27), the values of the experimental plasticity coefficient agree good with the theoretical ones determined by (18): when the amount of steel fibers increases from $V_f=1\%$ to $V_f=2\%$, the ratio between theoretical and experimental values remains almost constant and makes $\lambda_{obs}/\lambda_{teor} = 1.06\dots 1.08$. Expression (6) for the plasticity coefficient of non reinforced concrete in relation to the experimental value also slightly differs – up to 1.06 times.

The regression analysis of the performed experiments has showed that the plasticity coefficient (in relation to the quantity of steel fibers) can be described by the following empirical relationship:

$$\lambda_{sfrc,t} = \lambda_t \left(1.63 + 0.008V_f^{1,5} \right). \quad (28)$$

The nature of the plasticity coefficient for concrete in tension differs from that in compression (formula 27 and 28): the plasticity coefficient value is close to 0.90. This difference can be explained by the fact that the coefficients of elasticity and plasticity for concrete in tension approximately are equal $\nu_t \approx \lambda_t \approx 0.5$ (Залесов *et al.* 1988), i. e. elastic and plastic strains are of a similar value; moreover, they vary insignificantly depending on concrete class. Nonetheless, the plastic strain of SFRC tension is very heavy, and therefore $\nu_t \ll \lambda_t \approx 1$.

Experimental results presented in Table 4 show that the values of experimental and theoretical plasticity coefficient $\lambda_{obs}/\lambda_{cal}$ differ 1.08–1.10 times. Unfortunately, empirical relationships (27 and 28) are observed only in a few points (from our test results) and should be verified with a higher strength of concrete.

4. Conclusions

1. The proposed method analyzing the strength and strain of SFRC is based on the general principles of creating and modelling composites and following European standards for reinforced concrete but bearing in mind a direct evaluation of both elastic and plastic characteristics of the components (concrete and steel fibers).

2. The model gives an opportunity to determine tension and compression strengths, elasticity modulus and the main parameters – elasticity and plasticity coefficients of fiber concrete.

3. The results of experimental investigations and data obtained by other authors revealed a sufficient agreement between them and with those determined according to the proposed model. The ratios of theoretical and experimental values differ insignificantly and vary within the limits of 1.06–1.10.

4. The executed regression analysis of the results of experimental investigation offers a possibility of presenting simplified empirical formulas for determining the plasticity coefficient of SFRC at compression and tension

without deviation from the main additive law in the theory of composites.

5. This model may be used for the analysis of flexural SFRC members assuming normal stress distribution diagrams in tension and compression zones. For practical use, to verify the coefficient of plasticity using test results is recommended.

References

- Bareišis, J.; Kleiza, V. 2004. Research in the limiting efficiency of plastic deformation of multilayer tension (compression) bars, *Mechanics of Composite Materials* 40(2): 135–144. doi:10.1023/B:MOCM.0000025487.12111.c8
- Brandt, A. M. 2008. Fibre reinforced cement-based (FRC) composites after over 40 years of development in building and civil engineering, *Composite Structures* 86: 3–9. doi:10.1016/j.compstruct.2008.03.006
- Chalioris, C. E.; Karayannis, C. G. 2009. Effectiveness of the use of steel fibres on the torsional behaviour of flanged concrete beams, *Cement & Concrete Composites* 31: 331–341. doi:10.1016/j.cemconcomp.2009.02.007
- Chiaia, B.; Alessandro, P.; Fantilli, A. P.; Vallini, P. 2009. Combining fiber-reinforced concrete with traditional reinforcement in tunnel linings, *Engineering Structures* 31:1600–1606. doi:10.1016/j.engstruct.2009.02.037
- Fantilli, A.P.; Mihashi, H.; Vallini, P. 2009. Multiple cracking and strain hardening in fiber-reinforced concrete under uniaxial tension, *Cement and Concrete Research* 39: 1217–1229. doi:10.1016/j.cemconres.2009.08.020
- Harajli, M. H.; Maalouf, D.; Khatib, H. 1996. Effect of fibers on the punching shear strength of slab-column connections, *Cement & Concrete Composites* 17: 161–170. doi:10.1016/0958-9465(94)00031-5
- Johnston, C. D.; Zemp, R. W. 1991. Flexural fatigue performance of steel fibers – influence of fiber content, aspect ratio, and type, *ACI Materials Journal* 88(4): 374–383.
- Kanda, T.; Lin, Z.; Li, V. C. 2000. Modeling of Tensile Stress-Strain Relation of Pseudo Strain-Hardening Cementitious Composites, *ASCE Journal of Materials in Civil Engineering* 12(2): 147–156. doi:10.1061/(ASCE)0899-1561(2000)12:2(147)
- Kanda, T.; Li, V. C. 1999. Effect of Apparent Fiber Strength and Fiber-Matrix Interface on Crack Bridging in Cement Composites, *ASCE Journal of Engineering Mechanics* 125(3): 290–299. doi:10.1061/(ASCE)0733-9399(1999)125:3(290)
- Kang, Su-Tae; Lee, Yun; Park, Yon-Dong; Kim, Jin-Keun. 2010. Tensile fracture properties of an Ultra High Performance Fiber Reinforced Concrete (UHPFRC) with steel fiber, *Composite Structures* 92: 61–71. doi:10.1016/j.compstruct.2009.06.012
- Kasper, T.; Edvardsen, C.; Wittneben, G.; Neumann, D. 2008. Lining design for the district heating tunnel in Copenhagen with steel fibre reinforced concrete segments, *Tunneling and Underground Space Technology* 23: 574–587. doi:10.1016/j.tust.2007.11.001
- Laranjeira, F.; Molins, C.; Aguado, A. 2010. Predicting the pullout response of inclined hooked steel fibers, *Cement and Concrete Research* 40: 1471–1487. doi:10.1016/j.cemconres.2010.05.005
- Leung, C. K. Y.; Li, V. C. 1991. New Strength Based Model for the Debonding of Discontinuous Fibers in an Elastic

- Matrix, *Journal of Materials Science* 26: 5996–6010. doi:10.1007/BF01113875
- Li, V. C. 1992. A Simplified Micromechanical Model of Compressive Strength of Fiber Reinforced Cementitious Composites, *Journal of Cement and Concrete Composites* 14(2): 131–141. doi:10.1016/0958-9465(92)90006-H
- Li, V. C. 2002. Large Volume, High-Performance Applications of Fibers in Civil Engineering, *Journal of Applied Polymer Science* 83(2): 660–686. doi:10.1002/app.2263
- Li, V. C.; Wang, S. 2002. Flexural Behavior of GFRP Reinforced Engineered Cementitious Composites Beams, *ACI Materials Journal* 99(1): 11–21.
- Maalej, M.; Li, V. C., Hashida, T. 1995. Effect of Fiber Rupture on Tensile Properties of Short Fiber Composites, *ASCE Journal of Engineering Mechanics* 121(8): 903–913. doi:10.1061/(ASCE)0733-9399(1995)121:8(903)
- Maalej, M.; Li, V. C. 1994. Flexural Strength of Fiber Cementitious Composites, *ASCE Journal of Materials in Civil Engineering* 6(3): 390–406. doi:10.1061/(ASCE)0899-1561(1994)6:3(390)
- Maleki, S.; Mahoutian, M. 2009. Experimental and analytical study on channel shear connectors in fiber-reinforced concrete, *Journal of Constructional Steel Research* 65: 1787–1793. doi:10.1016/j.jcsr.2009.04.008
- Marčiukaitis, G. 1998. *Statybinių kompozitų kūrimo ir prognozavimo principai* [Design principals of composite materials]. Vilnius: Technika. 134 p.
- Meddah, M. S.; Bencheikh, M. 2009. Properties of concrete reinforced with different kinds of industrial waste fibre materials, *Construction and Building Materials* 23: 3196–3205. doi:10.1016/j.conbuildmat.2009.06.017
- Narayanan, R.; Darwish, I. Y. S. 1987. Use of Steel Fibers as Shear Reinforcement, *ACI Structural Journal* 84(3): 216–227.
- Nelson, P. K.; Li, V. C.; Kamada, T. 2002. Fracture Toughness of Micro-Fiber Reinforced Cement Composites, *ASCE Journal Materials in Civil Engineering* 14(5): 384–391. doi:10.1061/(ASCE)0899-1561(2002)14:5(384)
- Olivito, R. S.; Zuccarello, F. A. 2010. An experimental study on the tensile strength of steel fiber reinforced concrete, *Composites: Part B* 41: 246–255. doi:10.1016/j.compositesb.2009.12.003
- Özcan, D. M.; Bayraktar, A.; Sahin, A.; Haktanir, T.; Türker, T. 2009. Experimental and finite element analysis on the steel fiber-reinforced concrete (SFRC) beams ultimate behaviour, *Construction and Building Materials* 23: 1064–1077. doi:10.1016/j.conbuildmat.2008.05.010
- Pupurs, A.; Krasnikovs, A.; Pakrastinsh, L. 2006. Micro-mechanical stress-state analysis of fibre reinforced concrete (FRC), *Architecture and Construction Science* 2(7): 160–171.
- Soranakom, C.; Mobasher, B. 2008. Correlation of tensile and flexural responses of strain softening and strain hardening cement composites, *Cement & Concrete Composites* 30: 465–477. doi:10.1016/j.cemconcomp.2008.01.007
- Stang, H.; Li, V. C.; Krenchel, H. 1995. Design and Structural Applications of Stress-Crack Width Relations in Fiber Reinforced Concrete, *Journal of Materials and Structures* 28: 210–219. doi:10.1007/BF02473251
- Szmigiera, E. 2007. Influence of concrete and fibre concrete on the load-carrying capacity and deformability of composite steel-concrete columns, *Journal of Civil Engineering and Management* 8(1): 55–61.
- Šalna, R.; Marčiukaitis, G. 2010. Influence of fiber shape on the strength of steel fiber reinforced concrete. in *Selected papers of 10th International conference “Modern building materials, structures and techniques”*, Vilnius, II: 763–767. ISBN 978-9955-28-594-6
- Šalna, R.; Marčiukaitis, G. 2007. The influence of shear span ratio on load capacity of fibre reinforced concrete elements with various steel fibre volumes, *Journal of Civil Engineering and Management* 13(3): 209–215. ISSN 1392-3730.
- Wang, Y.; Backer, S.; Li, V. C. 1989. A Statistical Tensile Model of Fiber Reinforced Cementitious Composites, *Journal of Composites* 20(3): 265–274. doi:10.1016/0010-4361(89)90342-X
- Zhang, J.; Li, V. C. 2002. Effect of Inclination Angle on Fiber Rupture Load in Fiber Reinforced Cementitious Composites, *Composite Science and Technology* 62(6): 775–781.
- Zhang, J.; Stang, H.; Li, V. C. 2001. Crack Bridging for fibre reinforced concrete under fatigue tension, *International Journal of Fatigue* 23: 655–670
- Волков, В. И.; Газин, Э. М.; Бабекин, В. В. 2007. Инженерные методы проектирования фибробетонных конструкций [Volkov, V. I.; Gazin, E. M.; Babekin, V. V. Design methods of SFRC structures], *Бетон и железобетон* [Concrete and reinforced concrete] 4(547): 20–22.
- Залесов, А. С.; Кодыш, Э. Н.; Лемьш, Л. Л.; Никитин, И. К. 1988. *Расчет железобетонных конструкций по прочности, трещиностойкости и деформациям* [Zalsov, A. S.; Kodysch, E. N.; Lemysch, L. L.; Nikitin, I. K. Design of strength, cracking resistance and deformations of reinforced concrete structures]. Москва: Стройиздат. 320 с.
- Малмейстер, А. К.; Тамуж, В. П.; Тетерс, Г. А. 1980. *Сопротивление полимерных и композитных материалов* [Malmeister, A. K.; Tamuzh, V. P.; Teters, G. A. Resistance of polymeric and composite materials]. 3-е изд. Рига: Зинатне. 471 с.
- Рабинович, Ф. Н. 2004. *Композиты на основе дисперсно армированных бетонов. Вопросы теории и проектирования, технология, конструкции*: монография [Rabinovich, F. N. Composed structures, based on fiber reinforcement. The questions of theory, design, technology and structures]. Москва: АСВ. 506 с.
- Филипс, Д.; Харрис, Б. 1980. *Прочность, вязкость разрушения и усталостная выносливость полимерных композиционных материалов* [Filips, D.; Charris, B. Strength, failure and resistance of fatigue of polymeric composite materials]. Москва: Химия. 471 с.

BETONO, ARMUOTO PLIENINE DISPERSINE ARMATŪRA, STIPRIO IR DEFORMACIJŲ APSKAIČIAVIMO MODELIS

G. Marčiukaitis, R. Šalna, B. Jonaitis, J. Valivonis

Santrauka

Straipsnyje pasiūlytas betono, armuoto plienine dispersine armatūra, stiprio ir deformacijų skaičiavimo modelis, pagrįstas bendraisiais statybinių kompozitų kūrimo ir modeliavimo principais bei gelžbetonio normomis. Šiame modelyje skirtingai nuo daugelio kitų yra tiesiogiai įvertinamos tampriosios ir plastinės kompozito komponentų (betono ir plieninės dispersinės armatūros) savybės. Modelis leidžia apskaičiuoti betono, armuoto plienine dispersine armatūra, tempiamąjį ir gniuždomąjį stiprius, tamprumo modulį ir pagrindinius jo deformatyvumo parametrus – tamprumo ir plastiškumo koeficientus. Siūlomo modelio palyginimas su šio straipsnio ir kitų autorių atliktais eksperimentų duomenimis parodė, kad rezultatai sutampa. Teorinių ir eksperimentinių reikšmių santykiai skiriasi nedaug ir kinta nuo 1,06 iki 1,10. Šis modelis gali būti taikomas priimant įtempių pasiskirstymo diagramas apskaičiuojant lenkiamuosius, plienine dispersine armatūra armuotus betoninius elementus.

Reikšminiai žodžiai: kompozitas, betonas, plieninė dispersinė armatūra, betonas, armuotas plienine dispersine armatūra, deformacijų modulis, tampriosios ir plastinės deformacijos.

Gediminas MARČIUKAITIS. Prof., Dr Habil at the Department of Reinforced Concrete and Masonry Structures, Vilnius Gediminas Technical University (VGTU). PhD from Kaunas Polytechnical Institute in 1963. Research visit to the University of Illinois (1969). A Habilitated Doctor from Moscow Civil Engineering University in 1980. Professor (1982). The author and co-author of 5 monographs, 8 textbooks, 5 coursebooks and more than 300 scientific articles. Research interests: mechanics of reinforced concrete, masonry and layered structures, new composite materials, investigation and renovation of buildings.

Remigijus ŠALNA. A Doctor at the Department of Reinforced Concrete and Masonry Structures, Vilnius Gediminas Technical University, Lithuania. Research interests: punching shear strength of RC and SFRC slabs, investigation of buildings.

Bronius JONAITIS. Assoc. Prof., Dr at the Department of Reinforced Concrete and Masonry Structures, Vilnius Gediminas Technical University, Lithuania. The author and co-author of more than 50 scientific publications, 1 textbook, 3 coursebooks, 3 patented investigations. Research interests: theory of reinforced concrete behavior, masonry structures, strengthening of structures.

Juozas VALIVONIS. Prof., Dr at the Department of Reinforced Concrete and Masonry Structures, Vilnius Gediminas Technical University (VGTU). Publications: the author and co-author of more than 55 scientific publications, 4 textbooks, 5 coursebooks. Research interests: theory of reinforced concrete behavior, composite structures, reinforced concrete bridges.