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Corrigendum
Correction to the Paper: An Energy Dissipative Spatial Discretization for the Regularized Compressible Navier-Stokes-Cahn-Hilliard System of Equations (in *Math. Model. Anal.*, 25(1): 110–129, https://doi.org/10.3846/mma.2020.10577)

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Abstract. We correct the proof of Theorem 2 in the mentioned paper concerning finite-difference equilibrium solutions.

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In this note, we correct the proof of Theorem 2 (p. 120–121) in [1] following our recent paper [2]. Below we exploit the notation from [1].

We consider *the equilibrium solutions* $(\rho, \mathbf{u}, C) = (\rho_S, \mathbf{u}_S, C_S) \in H_X(\omega_h)$ with $\rho_S > 0$, $\mathbf{u}_S = 0$ and $0 < C_S(x) < 1$. For them, the following equations hold

$$\delta_i^* J_{iS} = 0, \tag{0.1}$$

$$s_l^*\big\{(s_l\rho)\big[\delta_l(G_h - \Phi) - (s_l\mu)\delta_l C\big]\big\} = 0,\ l = 1,2,3, \tag{0.2}$$

$$\delta_i^*\big[J_{iS} s_i C - M(s_i C)\delta_i\mu\big] = 0 \tag{0.3}$$

on ω_h (the summation from 1 to 3 is assumed over the repeated indices i), with

$$J_{lS} = -\big(s_l(\tau_0\rho)\big)\big[\delta_l(G_h - \Phi) - (s_l\mu)\delta_l C\big],\ l = 1,2,3, \tag{0.4}$$

Recall that $\tau_0 = \tau(\rho, \mathbf{u}, C)|_{\mathbf{u}=\mathbf{0}} > 0$, $\Phi \in H_X(\omega_h)$ is a given function and

$$G_h = \Psi'_{1\rho}(\rho, C) + \tfrac{1}{2}\lambda_1 s_i^*\big[(\delta_i C)^2\big], \quad \mu = \frac{1}{\rho}\big[\Psi'_{1C}(\rho, C) - \delta_i^*\big(\lambda_1(s_i\rho)\delta_i C\big)\big] \quad (0.5)$$

with the partial derivatives of a given function $\Psi_1(\rho, C)$.

Theorem 2. *The equilibrium solutions satisfy the following equations*

$$\Psi'_{1\rho}(\rho, C) + \tfrac{1}{2}\lambda_1 s_i^*\big[(\delta_i C)^2\big] - \mu_S C - \Phi \equiv G_h - \mu_S C - \Phi \equiv const, \quad (0.6)$$

$$\Psi'_{1C}(\rho, C) - \delta_i^*\big(\lambda_1(s_i\rho)\delta_i C\big) = \mu_S\rho, \quad \mu_S \equiv const \quad (0.7)$$

on ω_h, with the same functions Ψ_1 and Φ as in the differential case, see (2.12)–(2.13) in [1].

Proof. We first apply the known formula (for example, see formula (14) in [3])

$$\delta_l^*(J_{lS}s_l C) = (\delta_l^* J_{lS})C + s_l^*(J_{lS}\delta_l C)$$

and equation (0.1) and rewrite equation (0.3) as

$$s_i^*(J_{iS}\delta_i C) - \delta_i^*\big[M(s_i C)\delta_i\mu\big] = 0 \quad \text{on} \quad \omega_h. \quad (0.8)$$

We take the inner product in $H_X(\omega_h)$ of equations (0.1) and (0.8) respectively by $G_h - \Phi$ and μ and add the results. We apply both formulas (3.1) in [1] and get

$$-\big(J_{iS}, \delta_i(G_h - \Phi)\big)_{i*} + \big(J_{iS}\delta_i C, s_i\mu\big)_{i*} + \big(M(s_i C), (\delta_i\mu)^2\big)_{i*} = 0. \quad (0.9)$$

The substitution of expression (0.4) into (0.9) leads to the equality

$$\big(s_i(\tau_0\rho), \big[\delta_i(G_h - \Phi) - (s_i\mu)\delta_i C\big]^2\big)_{i*} + \big(M(s_i C), (\delta_i\mu)^2\big)_{i*} = 0.$$

Since $\tau_0\rho > 0$ and $M(s_i C) > 0$, it leads to the equalities

$$\delta_i(G_h - \Phi) - (s_i\mu)\delta_i C = 0, \quad \delta_i\mu = 0 \quad \text{on} \quad \omega_{i*,h}, \quad i = 1, 2, 3.$$

This first implies that $\mu \equiv \mu_S = \text{const}$ and then $G_h - \Phi - \mu_S C \equiv \text{const}$ on ω_h.

Consequently $J_{lS} = 0$, $l = 1, 2, 3$, and all the equations (0.1)–(0.4) are reduced to the two found constancy properties, i.e., according to definitions (0.5), to system (0.6)–(0.7) for ρ_S and C_S on ω_h.

Notice that above we have not required for the regularization parameter τ_0 to be constant as in [1].

References

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