



# Adaptive Stable Control of Manipulator System Based on Immersion and Invariance

Huapeng Wang<sup>a</sup>, Nan Jiang<sup>a</sup>, Ting Liu<sup>b</sup> and Yangyang Cao<sup>c</sup>

<sup>a</sup> *Criminal Investigation Police University of China*

No.83, Tawan Street, Huanggu District, 110000 Shenyang, China

<sup>b</sup> *College of Light Industry, Liaoning University*

No.66, Chongshan Middle Road, Huanggu District, 110000 Shenyang, China

<sup>c</sup> *College of Information Science and Engineering, Northeastern University*

No.3-11, Wenhua Road, Heping District, 110819 Shenyang, China

E-mail(*corresp.*): [jiangnan@ise.neu.edu.cn](mailto:jiangnan@ise.neu.edu.cn)

Received March 16, 2017; revised April 19, 2018; accepted April 20, 2018

**Abstract.** This work focused on the manipulator system containing uncertainties, and proposes an immersion and invariance (I&I) control strategy, in order to avoid the damage on the mechanical and the operation object caused by parameter uncertainty. A stable target system with lower dimension than the manipulator system was chosen to design the control law and estimation laws of uncertain parameters. Then finding an invariant and attractive manifold in state space with internal dynamics a copy of the desired closed-loop dynamics. Finally, design a control law that can steer the state of the system sufficiently close to the manifold. The immersion and invariance adaptive control does not rely on certainty equivalence. The whole uncertain parameter estimations are the sum of two terms. One is obtained by an iterative law like the traditional adaptive backstepping method. On the other hand, a nonlinear function is introduced. The role of this additional term makes the parameter estimations more flexible and effective. Lyapunov function is not necessary for the process of designing adaptive controllers. So immersion and invariance can effectively avoid the 'computing expansion' of backstepping method. Compared with the traditional adaptive methods, simulation results show that the proposed immersion and invariance adaptive controller can improve the system performance, including dynamic response, stability and accuracy of parameter estimations.

**Keywords:** manipulator, immersion and invariance, adaptive control, uncertainty, nonlinear control.

**AMS Subject Classification:** 93D21.

---

Copyright © 2018 The Author(s). Published by VGTU Press

This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

## 1 Introduction

The manipulator is a kind of mechanical device which is used for simulating the arm of human beings. In recent years, the study of manipulator technology has been making great progress. It is widely used in people's lives, such as industrial production, domestic service, space exploration. Manipulators have some special characteristics, including, strong coupling, nonlinear and parameter uncertainty because of manipulators complex structure. So controlling manipulators is subject to several difficult challenges. In brief, the study of manipulator control has an important and far-reaching significance.

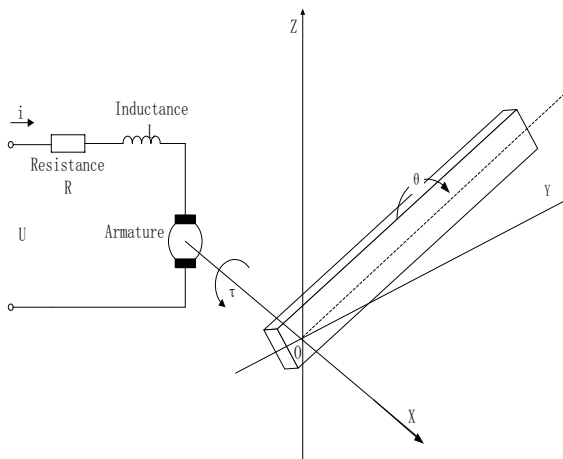
When the robot is in unknown situations, manipulators need not only a high precision control, but also the adaptability of unknown working situations. There are a lot of uncertainties in the dynamic characteristics of a manipulator, for example, structured characteristic parameters of payload and unstructured characteristics of friction and interference. Without adaptive estimation, the controller will face failure because of fixed parameters. Therefore, the influence of nonlinear and uncertain characteristics of the manipulator system should be considered when the controller is designed. To protect the controlled object and manipulator itself from damage of the uncertain parameters, it is a significant and challenging study field that takes more effective adaptive control strategy to estimate parameters.

During the past decade, several have been devoted to solving manipulators nonlinear control. Neural network, sliding-mode, fuzzy and backstepping are used to deal with the uncertain parameters. In [4], the input state linearization method is used to design a nonlinear controller for an omnidirectional mobile manipulator to realize its navigation and obstacle avoidance. In [13], an adaptive neural network compensator is proposed to improve the control performance of a macro-mini robotic manipulator, and it demonstrates that compared to the mini based on feedback linearization with the PID controller, vibration can be obviously suppressed using the adaptive compensator, and the steady state error is also gradually improved. In [16], for flexible joint manipulator, a comparison between PI and sliding mode control is proposed for the control to improve the performance in the presence of errors in the parameters identification, and it shows the high performances of sliding mode controller by converging the error to zero and guaranteeing the stability of the robot [14]. In [5,6,10], an adaptive backstepping design method is proposed to deal with the system uncertainty for the position tracking control of this robotic manipulator. But backstepping is likely to result in 'computing expansion'. Backstepping is one of the traditional adaptive methods which parameter estimations must follow the principles of certainty equivalence. Also Lyapunov function is necessary for the process of designing adaptive controllers. A large number of differential calculations will increase computation work. Moreover, it makes the whole designing process much more complicated. Therefore immersion and invariance adaptive control is proposed to solve the problems above in [7, 8, 11, 15]. Immersion and method is set up by an Italian scientist named A. Astolfi. A stable target system with lower dimension than the controlled system was chosen to design the control law and estimation laws of uncertain parameters in [1, 3, 12].

Since the manipulator system contains some uncertain parameters, immersion and invariance adaptive control method is made to design estimation laws of uncertain parameters. First, a stable target system with lower dimension than the manipulator system was chosen to design the control law and estimation laws of uncertain parameters. Then any trace of the controlled system is the target system in the immersion map. Finally, designing a control law that can steers the state of the system sufficiently close to the manifold [2]. Unlike most existing adaptive methods, the immersion and invariance adaptive control does not invoke certainty equivalence in [9]. Lyapunov function is unnecessary when the controller is designed in this method, which means differential operation is redundant, hence averting the 'computing expansion' of backstepping method effectively. Also the(I&I) adaptive control does not rely certainty equivalence principle. The whole uncertain parameters estimations are the sum of two terms. One is obtained by an update law like backstepping method. The other is an adequately chosen the nonlinear function. The role of this additional term makes the parameters estimations more flexible and effective.

## 2 Manipulator system model and control problems

We consider a one-link manipulator with the inclusion of motor dynamics. Its configuration is shown in Figure 1.



**Figure 1.** Configuration of one-link manipulator with the inclusion of motor dynamics.

It is assumed that the armature of the manipulator adopts a DC motor with the same performance parameters, and the reduction gear has the same reduction ratio  $n$ . The manipulator model is given by

$$\begin{aligned}
 D\ddot{q} + B\dot{q} + N \sin(q) &= \tau, \\
 M\dot{\tau} + H\tau &= u - K_m\dot{q},
 \end{aligned}$$

where  $q$ ,  $\dot{q}$ ,  $\ddot{q}$  are respectively the position, velocity and acceleration of the manipulator joint angle respectively,  $D$  meaning the inertia matrix,  $B$  representing the centripetal and coriolis force matrix,  $N$  standing for the friction and gravity matrix,  $\tau$  being the control moment.  $M = I_n L / n K_t$ ,  $H = I_n R / n K_t$ ,  $K_m = n K_e I_n$ .  $I_n$  is  $n$ -dimensional identity matrix,  $u$  is the voltage of motor dynamics,  $L$  is the inductance of motor dynamics,  $K_e$  is the resistance of motor dynamics,  $R$  is the back electromotive force constant of motor dynamics and  $K_t$  is the torque constant of motor dynamics.

When uncertainties are presented, the above equations can be expressed below

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1^2 \theta_1 + 0.2 \sin(0.1x_1), \\ \dot{x}_2 &= x_3 + x_2^2 \theta_2 - \frac{Bx_2}{D} - \frac{N}{D} \sin(x_1) + \frac{1-D}{D} x_3, \\ \dot{x}_3 &= \frac{1+x_3^2 \theta_3}{M} u - \frac{K_m}{M} x_2 - \frac{H}{M} x_3, \quad y = x_1, \end{aligned} \quad (2.1)$$

where  $x_1 = q$ ,  $x_2 = \dot{q}$ ,  $x_3 = \tau$ , when  $-\frac{B}{D} = B_0$ ,  $-\frac{N}{D} = N_0$ ,  $1 + \frac{1-D}{D} = D_0$ ,  $\frac{1}{M} = M_0$ ,  $\frac{K_m}{M} = K_{m0}$ ,  $\frac{H}{M} = H_0$ ,  $\theta_1, \theta_2, \theta_3$  are parameters of friction, motor factors and uncertainties caused by unknown grasping object. The system (2.1) can be rewritten as

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1^2 \theta_1 + 0.2 \sin(0.1x_1), \\ \dot{x}_2 &= D_0 x_3 + x_2^2 \theta_2 - B_0 x_2 - N_0 \sin(x_1), \\ \dot{x}_3 &= M_0 (1 + x_3^2 \theta_3) u - K_{m0} x_2 - H_0 x_3, \quad y = x_1. \end{aligned} \quad (2.2)$$

Next, an adaptive controller based on I&I method is design for system (2.2) and the designed I&I adaptive control law makes system's dynamic response more quickly and parameter estimations more accurate.

### 3 Design of I&I adaptive controller for manipulator system

If the system has stable equilibrium point, the control goal is to design the controller to make the system (2.2) global asymptotical stabilization approach the equilibrium point. The following process is divided into four steps to design a nonlinear controller.

#### 3.1 Target system

The manipulator system is a three-dimensional system, hence one of the lower dimension system is chosen as the target system. The two-dimensional system is presented in (3.1)

$$\begin{aligned} \dot{\xi}_1 &= \xi_2, \\ \dot{\xi}_2 &= -N_0 \sin(\xi_1) - B_0 \xi_2, \end{aligned} \quad (3.1)$$

where  $\xi_1, \xi_2 \in R$ ,  $N_0, B_0$  are parameters of manipulator system, and the system has a globally asymptotically stable equilibrium at  $\xi_*$  and  $x_* = \pi(\xi_*)$ .

### 3.2 Choose the immersion mapping

As is shown above, the controlled system and the target system have been chosen. Then immersion condition in the I&I method will give us the mapping  $\pi(\xi, \theta)$  and

$$\pi(\xi, \theta) = [\pi_1(\xi, \theta), \pi_2(\xi, \theta), \pi_3(\xi, \theta)]^T,$$

where  $\pi_1(\xi, \theta) = \xi_1$ ,  $\pi_2(\xi, \theta) = \xi_2$  and  $\pi_3(\xi, \theta)$  is an unknown mapping to be designed, i.e.

$$\pi(\xi, \theta) = \begin{bmatrix} \pi_1(\xi, \theta) \\ \pi_2(\xi, \theta) \\ \pi_3(\xi, \theta) \end{bmatrix} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \pi_3(\xi, \theta) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x.$$

According to the immersion condition ( $f(\pi(\xi, \theta), c(\xi, \theta), \theta) = \frac{\partial \pi}{\partial \xi} \alpha(\xi, \theta)$ ), the mapping relations between the controlled system and the target system are given by

$$\begin{aligned} \begin{bmatrix} \xi_2 + \xi_1^2 \theta_1 + 0.2 \sin(0.1 \xi_1) \\ D_0 \pi_3 + \xi_2^2 \theta_2 - B_0 \xi_2 - N_0 \sin(\xi_1) \\ M_0(1 + \pi_3^2)(c(\xi, \theta)) - k_{m0} \xi_2 - H_0 \pi_3 \end{bmatrix} &= \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \frac{\partial \pi_3(\xi, \theta)}{\partial \xi_1} \dot{\xi}_1 + \frac{\partial \pi_3(\xi, \theta)}{\partial \xi_2} \dot{\xi}_2 \end{bmatrix} \\ &= \begin{bmatrix} \xi_2 \\ -N_0 \sin(\xi_1) - B_0 \xi_2 \\ \frac{\partial \pi_3(\xi, \theta)}{\partial \xi_1} \xi_2 + \frac{\partial \pi_3(\xi, \theta)}{\partial \xi_2} [-N_0 \sin(\xi_1) - B_0 \xi_2] \end{bmatrix}. \end{aligned} \tag{3.2}$$

To calculate the second line of equation (3.2), then

$$\pi_3 = -\frac{1}{D_0} \xi_2^2 \theta_2. \tag{3.3}$$

Using equation (3.3) to derive differential relations, there is

$$\begin{cases} \frac{\partial \pi_3(\xi, \theta)}{\partial \xi_1} = 0, \\ \frac{\partial \pi_3(\xi, \theta)}{\partial \xi_2} = -\frac{2}{D_0} \xi_2 \theta_2. \end{cases} \tag{3.4}$$

By substituting equation (3.4) into the third line of equation (3.2), we get

$$c(\xi, \theta) = \frac{2N_0 D_0 \xi_2 \sin(\xi_1) \theta_2 + (2B_0 - H_0) D_0 \xi_2^2 \theta_2 + D_0^2 K_{m0} \xi_2}{M_0(D_0^2 + \xi_2^4 \theta_2^2 \theta_3)}.$$

### 3.3 Implicit manifold

The control objective is to keep all trajectories and drive the estimate asymptotically convergent to zero. We should make the manifold attractive and invariant.

$$\{x \in R^n | \phi(x, \theta) = 0\} = \{x \in R^n | x = \pi(\xi, \theta), \xi \in R^p\}.$$

Hence, we provide the implicit manifold

$$\phi(x, \theta) = x_3 - \pi_3([x_1, x_2]^T, \theta) = x_3 + \frac{1}{D_0} x_2^2 \theta_2.$$

### 3.4 Nonlinear adaptive control system design

Define the off-the-manifold  $\zeta = \phi(x, \hat{\theta})$ , then

$$\zeta = x_3 - \pi_3([x_1, x_2]^T, \theta) = x_3 + \frac{1}{D_0}x_2^2\theta_2. \quad (3.5)$$

For a more flexible design of the parameter estimation law introduce parameter  $\beta_i(x_1, x_2)$  and replace the unknown parameter  $\theta_i$  with  $\hat{\theta}_i + \beta_i(x_1, x_2)$ , where  $\dot{\hat{\theta}}_i = \omega_i$ ,  $\hat{\theta}_i$  is an estimated value of the unknown parameter  $\theta_i$ .

The partial differential of equation (3.5) is derived as

$$\begin{aligned} \dot{\zeta} &= \sum_{i=1}^3 \frac{\partial \phi}{\partial x_i} \dot{x}_i + \sum_{i=1}^3 \frac{\partial \phi}{\partial \hat{\theta}_i} \omega_i = \frac{1}{D_0} \frac{\partial \beta_2(x_1, x_2)}{\partial x_1} x_2^2 \dot{x}_1 + \frac{1}{D_0} [2\beta_2(x_1, x_2)x_2 \\ &\quad + \frac{\partial \beta_2(x_1, x_2)}{\partial x_2} x_2^2] \dot{x}_2 + \dot{x}_3 + \frac{1}{D_0} x_2^2 \omega_2 \\ &= \frac{1}{D_0} \frac{\partial \beta_2(x_1, x_2)}{\partial x_1} x_2^2 [x_2 + x_1^2 \theta_1 + 0.2 \sin(0.1x_1)] + \frac{1}{D_0} [2\beta_2(x_1, x_2)x_2 \\ &\quad + \frac{\partial \beta_2(x_1, x_2)}{\partial x_2} x_2^2] [D_0 x_3 + x_2^2 \theta_2 - B_0 x_2 - N_0 \sin(x_1)] \\ &\quad + M_0(1 + x_3^2 \theta_3)u - K_{m0}x_2 - H_0 x_3 + \frac{1}{D_0} x_2^2 \omega_2 = \frac{1}{D_0} \frac{\partial \beta_2(x_1, x_2)}{\partial x_1} x_2^2 \\ &\quad \times [x_2 + x_1^2 \theta_1 + 0.2 \sin(0.1x_1)] + \frac{1}{D_0} [2\beta_2(x_1, x_2)x_2 + \frac{\partial \beta_2(x_1, x_2)}{\partial x_2} x_2^2] \\ &\quad \times [D_0 x_3 + x_2^2 (\hat{\theta}_2 + \beta_2(x_1, x_2)) - B_0 x_2 - N_0 \sin(x_1)] \\ &\quad + M_0 [1 + x_3^2 (\hat{\theta}_3 + \beta_3(x_1, x_2))]u - K_{m0}x_2 - H_0 x_3 + x_2^2 \omega_2 / D_0. \end{aligned}$$

Let  $\dot{\zeta} = -\delta\zeta$ , and then the control law is designed as

$$\begin{aligned} u &= \frac{1}{M_0 [1 + x_3^2 (\hat{\theta}_3 + \beta_3(x_1, x_2))]} \left\{ -\delta\zeta + K_{m0}x_2 + H_0 x_3 - \frac{1}{D_0} x_2^2 \omega_2 \right. \\ &\quad - \frac{1}{D_0} [2\beta_2(x_1, x_2)x_2 + \frac{\partial \beta_2(x_1, x_2)}{\partial x_2} x_2^2] [D_0 x_3 + x_2^2 (\hat{\theta}_2 + \beta_2(x_1, x_2)) \\ &\quad - B_0 x_2 - N_0 \sin(x_1)] \\ &\quad \left. + \frac{1}{D_0} \frac{\partial \beta_2(x_1, x_2)}{\partial x_1} x_2^2 [x_2 + x_1^2 (\hat{\theta}_1 + \beta_1(x_1, x_2)) + 0.2 \sin(0.1x_1)] \right\}. \end{aligned}$$

The partial differential of the estimation errors  $z_i = \hat{\theta}_i - \theta_i + \beta_i(x_1, x_2)$  is

$$\begin{aligned} \dot{z}_i &= \omega_i + \frac{\partial \beta_i(x_1, x_2)}{\partial x_1} \dot{x}_1 + \frac{\partial \beta_i(x_1, x_2)}{\partial x_2} \dot{x}_2 \\ &= \omega_i + \frac{\partial \beta_i(x_1, x_2)}{\partial x_1} [x_2 + x_1^2 \theta_1 + 0.2 \sin(0.1x_1)] \\ &\quad + \frac{\partial \beta_i(x_1, x_2)}{\partial x_2} [D_0 x_3 + x_2^2 \theta_2 - B_0 x_2 - N_0 \sin(x_1)] \end{aligned}$$

$$\begin{aligned}
 &= \omega_i + \frac{\partial\beta_i(x_1, x_2)}{\partial x_1} \{x_2 + x_1^2[\hat{\theta}_1 + \beta_1(x_1, x_2) - z_1] + 0.2 \sin(0.1x_1)\} \\
 &+ \frac{\partial\beta_i(x_1, x_2)}{\partial x_2} \{D_0x_3 + x_2^2[\hat{\theta}_2 + \beta_2(x_1, x_2) - z_2] - B_0x_2 - N_0 \sin(x_1)\}, \quad (3.6)
 \end{aligned}$$

i.e.

$$\begin{aligned}
 \dot{z}_1 &= \omega_1 + \frac{\partial\beta_1(x_1, x_2)}{\partial x_1} \{x_2 + x_1^2[\hat{\theta}_1 + \beta_1(x_1, x_2) - z_1] + 0.2 \sin(0.1x_1)\} \\
 &+ \frac{\partial\beta_1(x_1, x_2)}{\partial x_2} \{D_0x_3 + x_2^2[\hat{\theta}_2 + \beta_2(x_1, x_2) - z_2] - B_0x_2 - N_0 \sin(x_1)\}, \\
 \dot{z}_2 &= \omega_2 + \frac{\partial\beta_2(x_1, x_2)}{\partial x_1} \{x_2 + x_1^2[\hat{\theta}_1 + \beta_1(x_1, x_2) - z_1] + 0.2 \sin(0.1x_1)\} \\
 &+ \frac{\partial\beta_2(x_1, x_2)}{\partial x_2} \{D_0x_3 + x_2^2[\hat{\theta}_2 + \beta_2(x_1, x_2) - z_2] - B_0x_2 - N_0 \sin(x_1)\}, \\
 \dot{z}_3 &= \omega_3 + \frac{\partial\beta_3(x_1, x_2)}{\partial x_1} \{x_2 + x_1^2[\hat{\theta}_1 + \beta_1(x_1, x_2) - z_1] + 0.2 \sin(0.1x_1)\} \\
 &+ \frac{\partial\beta_3(x_1, x_2)}{\partial x_2} \{D_0x_3 + x_2^2[\hat{\theta}_2 + \beta_2(x_1, x_2) - z_2] - B_0x_2 - N_0 \sin(x_1)\}.
 \end{aligned}$$

Select  $\omega_i$  as follows

$$\begin{aligned}
 \omega_i &= -\frac{\partial\beta_i(x_1, x_2)}{\partial x_1} \{x_2 + x_1^2[\hat{\theta}_1 + \beta_1(x_1, x_2)] + 0.2 \sin(0.1x_1)\} \\
 &- \frac{\partial\beta_i(x_1, x_2)}{\partial x_2} \{D_0x_3 + x_2^2[\hat{\theta}_2 + \beta_2(x_1, x_2)] - B_0x_2 - N_0 \sin(x_1)\}.
 \end{aligned}$$

Substituting  $\omega_i$  into (3.6), we have

$$\dot{z}_i = -\frac{\partial\beta_i(x_1, x_2)}{\partial x_1} x_1^2 z_1 - \frac{\partial\beta_i(x_1, x_2)}{\partial x_2} x_2^2 z_2,$$

i.e.

$$\begin{aligned}
 \dot{z}_1 &= -\frac{\partial\beta_1(x_1, x_2)}{\partial x_1} x_1^2 z_1 - \frac{\partial\beta_1(x_1, x_2)}{\partial x_2} x_2^2 z_2, \\
 \dot{z}_2 &= -\frac{\partial\beta_2(x_1, x_2)}{\partial x_1} x_1^2 z_1 - \frac{\partial\beta_2(x_1, x_2)}{\partial x_2} x_2^2 z_2, \\
 \dot{z}_3 &= -\frac{\partial\beta_3(x_1, x_2)}{\partial x_1} x_1^2 z_1 - \frac{\partial\beta_3(x_1, x_2)}{\partial x_2} x_2^2 z_2.
 \end{aligned}$$

Let  $V = (z_1^2 + z_2^2 + z_3^2)/2$ . After taking the derivative of  $V$ , the following expression is obtained

$$\begin{aligned}
 \dot{V} &= z_1\dot{z}_1 + z_2\dot{z}_2 + z_3\dot{z}_3 \\
 &= -z_1\left(\frac{\partial\beta_1(x_1, x_2)}{\partial x_1} x_1^2 z_1 + \frac{\partial\beta_1(x_1, x_2)}{\partial x_2} x_2^2 z_2\right) - z_2\left(\frac{\partial\beta_2(x_1, x_2)}{\partial x_1} x_1^2 z_1\right)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{\partial \beta_2(x_1, x_2)}{\partial x_2} x_2^2 z_2) - z_3 \left( \frac{\partial \beta_3(x_1, x_2)}{\partial x_1} x_1^2 z_1 + \frac{\partial \beta_3(x_1, x_2)}{\partial x_2} x_2^2 z_2 \right) \\
 = & -z_1^2 x_1^2 \frac{\partial \beta_1(x_1, x_2)}{\partial x_1} - z_1 z_2 x_2^2 \frac{\partial \beta_1(x_1, x_2)}{\partial x_2} - z_1 z_2 x_1^2 \frac{\partial \beta_2(x_1, x_2)}{\partial x_1} \\
 & - z_2^2 x_2^2 \frac{\partial \beta_2(x_1, x_2)}{\partial x_2} - z_1 z_3 x_1^2 \frac{\partial \beta_3(x_1, x_2)}{\partial x_1} - z_2 z_3 x_2^2 \frac{\partial \beta_3(x_1, x_2)}{\partial x_2}.
 \end{aligned}$$

In order to ensure  $\dot{V} \leq 0$ , we know

$$\begin{aligned}
 \frac{\partial \beta_1(x_1, x_2)}{\partial x_1} & \geq 0; \quad \frac{\partial \beta_1(x_1, x_2)}{\partial x_2} = 0; \quad \frac{\partial \beta_3(x_1, x_2)}{\partial x_1} = 0, \\
 \frac{\partial \beta_2(x_1, x_2)}{\partial x_2} & \geq 0; \quad \frac{\partial \beta_2(x_1, x_2)}{\partial x_1} = 0; \quad \frac{\partial \beta_3(x_1, x_2)}{\partial x_2} = 0.
 \end{aligned}$$

Choose the adjustment function as follows

$$\begin{cases} \beta_1(x_1, x_2) = x_1^3, \\ \beta_2(x_1, x_2) = x_2^3, \\ \beta_3(x_1, x_2) = 0. \end{cases}$$

We can obtain

$$\begin{cases} \omega_1 = -3x_1^2 \{x_2 + x_1^2[\hat{\theta}_1 + x_1^3] + 0.2 \sin(0.1x_1)\}, \\ \omega_2 = -3x_2^2 \{D_0 x_3 + x_2^2[\hat{\theta}_2 + x_2^3] - B_0 x_2 - N_0 \sin(x_1)\}, \\ \omega_3 = 0. \end{cases}$$

In conclusion, the control law is designed as

$$\begin{aligned}
 u = & \frac{1}{M_0(1 + x_3^2 \hat{\theta}_3)} \left\{ -\delta [x_3 + \frac{1}{D_0} x_2^2 (\hat{\theta}_2 + x_2^3)] + K_{m0} x_2 + H_0 x_3 \right. \\
 & \left. - \frac{2}{D_0} x_2^4 [D_0 x_3 + x_2^2 (\hat{\theta}_2 + x_2^3) - B_0 x_2 - N_0 \sin(x_1)] \right\}.
 \end{aligned}$$

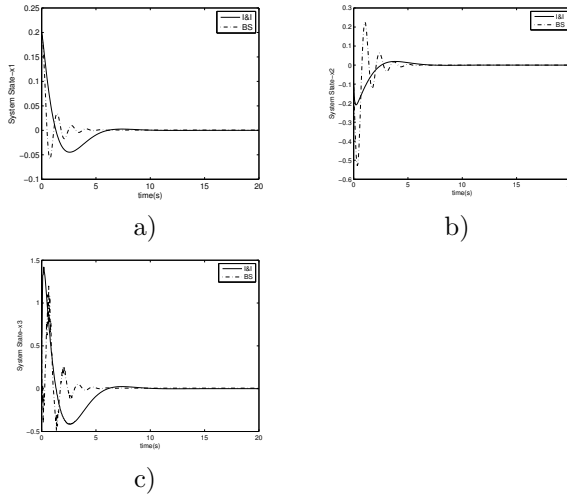
### 4 Simulation and analysis

The stability of the manipulator system is discussed by analyzing the system state responses under the Simulink of MATLAB environment. The system performance is shown in the process of simulation, including dynamic response, stability and accuracy of parameter estimation. To prove the effectiveness and superiority of the proposed I&I adaptive controller, the traditional backstepping adaptive controller is defined. In both cases, the manipulator system parameters are chosen as follows:

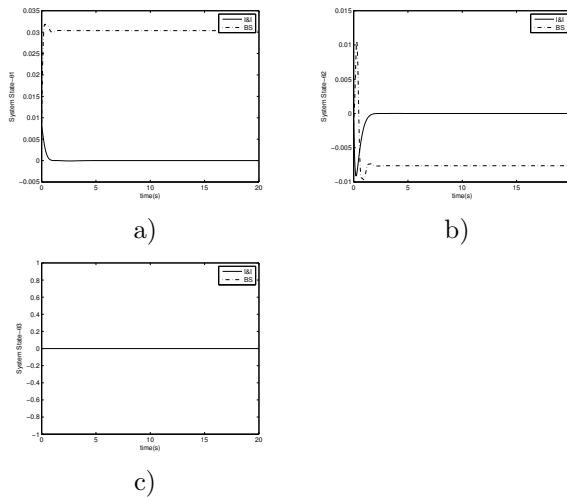
$$D = 1, \quad M = 0.05L, \quad B = 1, \quad K_m = 10, \quad H = 0.5\Omega, \quad N = 10, \quad \delta = 1.$$

Simulation results in Figures 2-3 show the comparison result between the proposed immersion and invariance I&I adaptive controller and the traditional backstepping adaptive method (BS).





**Figure 2.** Dynamic response of a)  $x_1$ , b)  $x_2$ , c)  $x_3$ .



**Figure 3.** Parameter estimation of a)  $\theta_1$ , b)  $\theta_2$ , c)  $\theta_3$ .

Under the backstepping adaptive method, the dynamic responses fluctuate strongly and tend to be stable after 10–11 seconds. For the I&I adaptive method it can be clearly seen that the system dynamic responses fluctuate smoothly and converge to be stable state more rapidly after about 5–7 seconds. Simulation results show the good performance of the proposed I&I adaptive controller with overshoot effectively suppressed and the settling time apparently reduced. As a result, the proposed I&I adaptive controller improves the system properties, including dynamic response, stability and accuracy of parameter estimation.

## 5 Conclusions

This paper investigated an adaptive controller for manipulator with uncertain parameters based on immersion and invariance (I&I) method to improve system dynamic quality. Conventional linearization methods have limitations, which only can guarantee the stability of certain conditions. Although adaptive backstepping method completely retained the nonlinear property of the system, the design process was based on Lyapunov function, which would cause inflation problems of the calculation in calculation due to the increase of the system order. The proposed I&I method effectively avoids the above shortcomings because it does not need the certainty equivalence. The controller design relies on finding a mapping relationship between the manipulator system and the target system, so the controller expression is more concise and it will be more popular in engineering application. Compared with the traditional adaptive methods, simulation results show that the proposed I&I controller not only makes the system overshoot effectively suppressed, but also apparently reduces the settling time.

## Acknowledgement

The first author's work was supported by Science Foundation for Doctorate Research of Criminal Investigation Police University of China under Grant D2017022, D2017021, National Natural Science Foundation of China, under grant 61304021 and 61233002, Key Special Program of the Ministry of Science and Technology of China under Grant 2017YFC0821000, Educational Commission of Liaoning Province under Grant L2015198, Science Foundation for Doctorate Research of Liaoning Province under Grant 201601091.

## References

- [1] A. Astolfi, D. Karagiannis and R. Ortega. Towards applied nonlinear adaptive control. *Annual Reviews in Control*, **32**(2):136–148, 2008. <https://doi.org/10.1016/j.arcontrol.2008.08.003>.
- [2] A. Astolfi, D. Karagiannis and R. Ortega. *Nonlinear and adaptive control with applications*. Springer-Verlag London Limited, London, 2007.
- [3] A. Astolfi and R. Ortega. Immersion and invariance: a new tool for stabilization and adaptive control of nonlinear systems. *IEEE Transactions on Automatic Control*, **48**(4):590–606, 2003. <https://doi.org/10.1109/TAC.2003.809820>.
- [4] S. Djebbarani, A. Benali and F. Abdessamed. Modelling and feedback control of an omni-directional mobile manipulator. In *The 7th IEEE Conference on Automation Science and Engineering, Trieste, Italy, 2011*, pp. 785–791. IEEE, 2011. <https://doi.org/10.1109/CASE.2011.6042441>.
- [5] Y. Hsiao, H. Tu and W. Hung. Sliding backstepping control design for robotic manipulator systems with motor dynamics. In *The 11th IEEE International Conference on Control Automation, Taichung, Taiwan, 2014*, pp. 667–672. IEEE, 2014. <https://doi.org/10.1109/ICCA.2014.6870999>.

- [6] N. Jiang, S. Li and T. Liu. Nonlinear large disturbance attenuation controller design for power systems with STATCOM. *Applied Mathematics and Computation*, **219**(20):10378–10386, 2013. <https://doi.org/10.1016/j.amc.2013.04.011>.
- [7] IU Khan and R Dhaouadi. Nonlinear reduced order observer design for elastic drive systems using invariant manifolds. In *2015 IEEE International Conference on Mechatronics, Nagoya, Japan, 2015*, pp. 58–63. IEEE, 2015. <https://doi.org/10.1109/ICMECH.2015.7083948>.
- [8] X. Liu, G. Hong and H. Luo. The immersion and invariance adaptive control for a class of linear motor systems with its application. In *The 27th Chinese Control and Decision Conference, Qingdao, China, 2015*, pp. 1502–1507. IEEE, 2015. <https://doi.org/10.1109/CCDC.2015.7162157>.
- [9] Z. Liu, X. Tan and R. Yuan. Nonlinear adaptive control for hypersonic vehicles via immersion and invariance. In *The 32th Chinese Control Conference, Xian, China, 2013*, pp. 2951–2956. IEEE, 2013.
- [10] R. Mei, X. Wu and S. Jiang. Robust adaptive backstepping control for a class of uncertain nonlinear systems based on disturbance observers. *China Information Sciences*, **53**(6):1201–1215, 2010. <https://doi.org/10.1007/s11432-010-3116-s>.
- [11] P. Rapp, M. Klunder and O. Sawodny. Nonlinear adaptive and tracking control of a pneumatic actuator via immersion and invariance. In *IEEE 51st Annual Conference on Decision and Control, Maui, HI, USA, 2012*, pp. 4145–4151. IEEE, 2012. <https://doi.org/10.1109/CDC.2012.6426396>.
- [12] I. Sarras. On the stabilization of nonholonomic mechanical systems via immersion and invariance. *IFAC Proceedings Volumes*, **44**(1):7227–7232, 2011. <https://doi.org/10.3182/20110828-6-IT-1002.01537>.
- [13] Y. Wu, C. Lai and S. Chen. An adaptive neural network compensator for decoupling of dynamic effects of a macro-mini manipulator. In *2015 IEEE International Conference on Advanced Intelligent Mechatronics, Busan, South Korea, 2015*, pp. 1427–1432. IEEE, 2015. <https://doi.org/10.1109/AIM.2015.7222741>.
- [14] C. Zhang, A. Zhang and H. Zhang. RBF neural networks sliding mode controller design for static var compensator. In *The 34th Chinese Control Conference, Hangzhou, China, 2015*, pp. 3501–3506. IEEE, 2015. <https://doi.org/10.1109/ChiCC.2015.7260179>.
- [15] B. Zhao, B. Xian and Y. Zhang. Nonlinear robust adaptive tracking control of a quadrotor UAV via immersion and invariance methodology. *IEEE Transactions on Industrial Electronics*, **62**(5):2891–2902, 2015. <https://doi.org/10.1109/TIE.2014.2364982>.
- [16] L. Zouari, H. Abid and M. Abid. Comparative study between PI and sliding mode controllers for flexible joint manipulator driving by brushless DC motor. In *The 14th International Conference on Sciences and Techniques of Automatic Control and Computer Engineering, Sousse, Tunisia, 2013*, pp. 294–299. IEEE, 2013. <https://doi.org/10.1109/STA.2013.6783146>.