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Parameter Control of Optical Soliton in One-Dimensional Photonic Crystal

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Abstract. The paper deals with finding of soliton solution for Schrödinger equation with periodic linear and nonlinear properties of medium in 1D case. Such structure is named as photonic crystal. To find soliton solution the corresponding problem for finding of eigenfunctions and eigenvalues is formulated. Iterative process is proposed for solution of this problem. Using the technique of continuation on parameter we investigate a dependence of soliton location on its maximum intensity, on ratio between light frequency and frequency of structure, on ratio between dielectric permittivity of alternating linear and nonlinear layers and on position between centre of initial distribution of eigenfunction and center of considered photonic structure area. The results of this paper confirm the features of soliton self-formation investigated early in our papers [37, 38, 39, 40, 41, 42], in which one considered a propagation of femtosecond laser pulse through nonlinear layered structure.

Keywords: photonic crystal, nonlinear Schrödinger equation, eigenfunctions, eigenvalues, iterative process, optical soliton.

AMS Subject Classification: 65M06; 65M22.

1 Introduction

Interaction of laser pulse with photonic crystal (PC), which is a periodic structure, and with other periodic structure is one of the actual problems of modern laser physics [3, 11, 12, 13, 14, 17, 18, 26, 28, 45, 46]. Among the various problems of such interaction the light localization and soliton formation are very attractive: there are many papers dealing with these phenomena [1, 8, 10, 15, 16, 19, 20, 21, 22, 23, 24, 29, 32, 37, 38, 39, 40, 41, 42, 44, 47].

For example, light localization has a great interest for various applications in all-optical switchers and optical data storage devices.

As it is well-known, there are various physical mechanisms of a light localization in periodic structures. In our recent papers [37, 38, 39, 40, 41, 42], we reported about a possibility of nonlinear light energy localization in 1D PC. It takes place due to soliton sub-pulses formation in nonlinear (self-focusing) layers while the laser pulse propagates through the PC. Sub-pulses propagate inside the nonlinear layer and totally reflect from its boundaries (neighboor layers are linear or defocusing ones). As it was shown in [42], these self-formed sub-pulses are solitons and there is a good agreement with well-known analytical soliton solution for a nonlinear cubic medium.

Another known effect of the light localization is Anderson one [13] in linear periodic structure. It takes place for disordered PC. In practice, it means a bad quality of the PC. So, for the case of layered structure it means that the thickness of layers changes randomly from layer to layer. Hence, a local band-gap of PC for various frequencies appears. Such effect takes place for the structure with big number of layers (more than 30) in a contrast to the nonlinear localization which takes place even for the PC with several layers [41]. It should be also mentioned that for the observation of the Anderson localization it is necessary that the carrying frequency of an input optical pulse is near the band-gap or stronger PC layers thickness fluctuations take place. Under such conditions the light energy localises in a part of PC, but not in a definite layer as it realises at the nonlinear light localization. It is very important that nonlinear light localization can occur as well for light frequencies, which belong to the frequency range of a transparency of the linear PC. In this case it is necessary that an input pulse intensity must be greater than its critical value [37], which decreases in disordered layered structure [37].

At investigation of soliton formation in a nonlinear 1D PC has shown realization of unmoving spatial distribution of optical radiation, which is located near the boundary of layers. From mathematical point of view, such mode of laser light propagation is a soliton mode. Such modes of optical radiation would be very important in many applications, e.g for construction of optical data storage devices. Hence, we investigate below a dependence of soliton formation on various parameters of optical pulse interaction with layered structure and an opportunity for controlling the soliton displacement in PC and their parameters.

2 Statement of Problem for Laser Pulse Propagation in PC

As it is well-known, femtosecond pulse propagation in 1D photonic crystal is governed by the following wave equation:

$$\frac{\partial^2 E(z,t)}{\partial z^2} - \frac{n^2(z)}{c^2}\frac{\partial^2 E(z,t)}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}P_{nl}, \quad 0 < t < L_t, \;\; 0 < z < L_z, \qquad (2.1)$$

where $P_{nl} = \chi^{(3)}|E|^2 E$, and $E(z,t)$ is the electric field strength; z is a coordinate, along which the pulse propagates, and L_z its maximum value; $n(z) =$

$\sqrt{\varepsilon(z)}$ is a medium refractive index; t is time and L_t is time interval, within which a pulse propagation is analysed; c is a light velocity and $\chi^{(3)}$ is a cubic susceptibility of medium.

To simplify a mathematical description of an optical pulse propagation we transform the wave equation (2.1) to Schrödinger equation with respect to complex amplitude $A(z,t)$, which is slowly varying in time and fast varying on the spatial coordinate. It means that we do not make a choice of light propagation direction for incident wave on PC as it is usually used for consideration of this problem [27]. The reason of choosing this approach concludes in self-validity of certain invariant (conservation law that is named by us as spectral invariant [33, 34]) at the solution of corresponding Schrödinger equation. If we apply the standard approach [27] it is necessary to control this invariant additionally. Hence, an accuracy of computation increases and computation algorithm for our approach is more simple and stable in comparison with computation algorithms, that are based on well-known approach. Thus, the electric field strength and nonlinear medium response can be written in the form

$$E(z,t) = \frac{1}{2}E_0(A(z,t)e^{-i\omega t} + c.c.), \quad P_{nl} = \frac{1}{8}\chi^{(3)}\left|A\right|^2 E_0^3(A(z,t)e^{-i\omega t} + c.c.).$$

Here $c.c.$ denotes a conjugation of complex function, E_0 is amplitude of the electric field strength. Assuming the linear relationship between the wave number k and frequency of light ω, as it is frequently done for a considered class of problems [27], and using the same coordinates' notations for convenience, one can get from wave equation (2.1) the following nonlinear Schrödinger equation, which is written in dimensionless variables

$$\varepsilon(z)\frac{\partial A}{\partial t} + iD\frac{\partial^2 A}{\partial z^2} + i\beta\left[\varepsilon(z) + \alpha(z)\left|A\right|^2\right]A = 0, \; 0 < t \le L_t, \; 0 < z < L. \quad (2.2)$$

The following parameters are used above: $D = -\frac{1}{4\pi\Omega}$, $\beta = -\pi\Omega$, $\Omega = \frac{\omega}{\omega_{str}}$, $L = L_z/\lambda_0$,

$$\varepsilon(z) = \begin{cases} 1, & 0 \le z \le L_0, \\ \varepsilon_1, & 0 \le z - L_0 - (d_1+d_2)(j-1) \le d_1, 1 \le j < N_{str}+1, \\ \varepsilon_2, & 0 \le z - L_0 - (d_1+d_2)(j-1) - d_1 \le d_2, 1 \le j < N_{str}, \\ \varepsilon_3, & L_0 + (d_1+d_2)N_{str} + d_1 \le z \le L. \end{cases}$$

$$\alpha(z) = \begin{cases} 0, & 0 \le z \le L_0, \\ \alpha_1, & 0 \le z - L_0 - (d_1+d_2)(j-1) \le d_1, 1 \le j < N_{str}+1, \\ \alpha_2, & 0 \le z - L_0 - (d_1+d_2)(j-1) - d_1 \le d_2, 1 \le j < N_{str}, \\ 0, & L_0 + (d_1+d_2)N_{str} + d_1 \le z \le L. \end{cases}$$

Function $\varepsilon(z)$ describes a dielectric permittivity of medium before PC, in PC layers and in substrate (medium after PC). From physical point of view it follows that $\varepsilon(z) > 0$. It should be noted that the case $\varepsilon = 1$ in any layer of PC or substrate corresponds to its measurement in units of permittivity for medium before the PC. Let's notice that the photonic crystal begins from layer with dielectric permittivity ε_1 and a dielectric permittivity of the last layer of PC is equal to ε_1 as well. In practice the substrate follows after this layer. Nevertheless, below we will be interested in a location of soliton inside the photonic crystal or inside some layers of photonic crystal. Hence, under the investigation of soliton formation we will consider only the domain of photonic crystal.

Above, ω_{str} is a frequency of periodic structure: $\omega_{str} = 2\pi c/\lambda_0$, $\lambda_0 = d_1\sqrt{\varepsilon_1} + d_2\sqrt{\varepsilon_2}$; d_1, d_2 and ε_1, ε_2 are thickness and permittivity of alternating layers; N_{str} is number of layers pairs; ε_3 is substrate permittivity. L_0 denotes dimensionless distance before PC; L is the normalised length of considering domain (it includes a distance before PC, length of layered structure, and substrate length), parameters α_1, α_2 characterise the cubic nonlinearity of alternating layers. Schematically the distribution of nonlinear coefficient α is shown in Fig. 1 for the domain of photonic crystal.

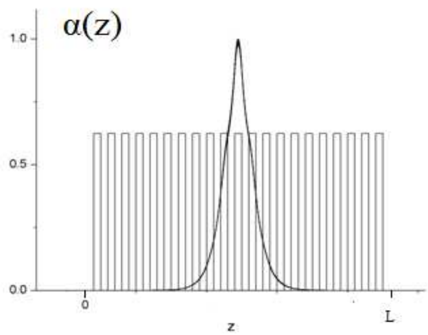

Figure 1. Spatial distribution of nonlinearity of medium and initial spatial distribution of optical intensity inside the PC.

In this figure L_0 is equal to zero and L coincides with the end of photonic crystal. It should be noticed that the initial condition for (2.2) becomes

$$A\Big|_{t=0} = A_0(z)e^{i2\pi\Omega(z-L_c)}\,, \; A_0(z) = e^{-\frac{(z-L_c)^2}{a^2}}. \qquad (2.3)$$

Here L_c is a coordinate of laser pulse centre on z axis and a characterises pulse duration. Spatial distribution (2.3) corresponds to one of the physical situations taking place in practice. For the first situation the laser pulse falls on the PC and propagates in the direction, which is transverse to layered structure. Then, the laser pulse is located before the PC.

The second situation corresponds to a propagation of laser pulse along the layers of PC. In this case the laser pulse is located inside a PC (Fig.1). Then, a parameter Ω means a perturbation of propagation direction. It does not equal to zero for inclined incidence of laser beam with respect of direction along which the layers are placed. An investigation of a stability for the laser pulse propagation with respect to transverse perturbation of propagation direction is reasonability made by the usage of so-called nonreflecting (transparent) boundary conditions [30, 31]:

$$\begin{aligned} &\frac{\partial A}{\partial t} - \frac{1}{\sqrt{\varepsilon(z)}}\frac{\partial A}{\partial z} + i2\beta A = 0,\ t > 0,\ z = 0, \\ &\frac{\partial A}{\partial t} + \frac{1}{\sqrt{\varepsilon(z)}}\frac{\partial A}{\partial z} + i2\beta A = 0,\ t > 0,\ z = L. \end{aligned} \tag{2.4}$$

It should be stressed that the development of the nonreflecting boundary conditions is a very modern problem. There are many papers on this subject. For example, we mention two review papers [2, 4]. Our approach at creation of nonreflecting boundary conditions concludes in developing of such conditions which allow to create a two- layer conservative finite-difference scheme for nonlinear Schrödinger equations. Let's note that the conservative finite-difference scheme for nonlinear Schrödinger equations which describe various nonlinear optics problems were discussed recently in [5, 6] as well. Nevertheless, we use the zero value boundary conditions as well under consideration of the eigenfunction (EF) and eigenvalues (EV) problem that is formulated below. These conditions look like $A|_{z=L_1} = 0$, $A|_{z=L_2} = 0$, here L_1, L_2 are coordinates of domain, which either coincide with the domain of photonic crystal or are located inside the photonic crystal if we want to find the solution that is located in some layer of PC.

For problem (2.2), (2.4) two invariants (conservation laws) [31] take place:

$$\begin{aligned} I_1(t) &= \int_0^L \varepsilon(z)|A|^2 dz - 2D\int_0^t \mathrm{Im}\Big(\Big(\frac{\partial A}{\partial z}A^*\Big)\Big|_{z=L} + \Big(\frac{\partial A^*}{\partial z}A\Big)\Big|_{z=0}\Big)d\eta = const, \\ I_3(t) &= \int_0^L \Big(-D\Big|\frac{\partial A}{\partial z}\Big|^2 + \beta\Big[\varepsilon(z) + 0.5\alpha(z)|A|^2\Big]|A|^2\Big)dz \\ &\quad + 2D\int_0^t \mathrm{Re}\Big(\Big(\frac{\partial A}{\partial z}\frac{\partial A^*}{\partial \eta}\Big)\Big|_{z=L} - \Big(\frac{\partial A}{\partial z}\frac{\partial A^*}{\partial \eta}\Big)\Big|_{z=0}\Big)d\eta = const. \end{aligned} \tag{2.5}$$

3 Eigenfunction Problem

To find EFs of equation (2.2) its solution is presented as $A = u(z)e^{-i\lambda t}$. After substituting this function into (2.2), the following boundary value problem is derived:

$$-\frac{1}{4\pi\Omega\varepsilon(z)}\frac{d^2u}{dz^2} - \pi\Omega\frac{\alpha(z)}{\varepsilon(z)}u|u|^2 = \tilde{\lambda}u, \quad \tilde{\lambda} = \lambda + \pi\Omega \tag{3.1}$$

$$u(L_1) = 0,\ \ u(L_2) = 0. \tag{3.2}$$

It should be noted that in (3.1) we introduce the shifted EVs $\tilde{\lambda}$. It does not restrict the problem because we add the same positive number to all EVs. Generally speaking, equation (3.1) can possess both real and complex EVs. However, taking into account that the function $\varepsilon(z)$ is a real one, it can be shown that EVs of equation (3.1) are real. Actually, if we multiply equation (3.1) by $\varepsilon(z)u^*$ (symbol star means a conjugation of complex function) and integrate it from L_1 to L_2, then after using boundary conditions (3.2), we will obtain the relation:

$$\int_{L_1}^{L_2} \Big(\frac{1}{4\pi\Omega}\Big|\frac{du}{dz}\Big|^2 - \pi\Omega\alpha(z)|u|^4\Big)dz = \tilde{\lambda}\int_{L_1}^{L_2} \varepsilon(z)|u|^2 dz.$$

Taking into account that the integrals on the left and right sides are real, we will conclude that $\tilde{\lambda}$ is real.

Let us introduce the uniform grid

$$\omega_z = \{z_n = nh_z + L_1,\ \ n = \overline{0, N_z},\ L_2 - L_1 = h_z N_z\}$$

and define grid functions u_h, α_h, ε_h on ω_z: $u_n = u(z_n)$, $\alpha_{h,n} = \alpha(z_n)$, $\varepsilon_{h,n} = \varepsilon(z_n)$ and the difference Laplace operator $u_{\bar{z}z,n} = (u_{n+1} - 2u_n + u_{n-1})/h_z^2$. Then the finite-difference scheme for equation (3.1) and boundary conditions (3.2) are written as:

$$-\frac{1}{4\pi\Omega\varepsilon_{h,n}}u_{\bar{z}z,n} - \pi\Omega\frac{\alpha_{h,n}}{\varepsilon_{h,n}}u_n|u_n|^2 = \tilde{\lambda}u_n, \quad n = \overline{1, N_z - 1},$$
$$u_0 = 0, \quad u_{N_z} = 0.$$

Because the above equations are nonlinear, the following iterative process is used:

$$\begin{aligned} &-\frac{1}{4\pi\Omega\varepsilon_{h,n}}\overset{s+1}{u}_{\bar{z}z,n} - \pi\Omega\frac{\alpha_{h,n}}{\varepsilon_{h,n}}\overset{s+1}{u}_n|\overset{s}{u}_n|^2 = \overset{s+1}{\tilde{\lambda}}\,\overset{s+1}{u}_n, \quad n = \overline{1, N_z - 1}, \\ &\overset{s+1}{u}_0 = \overset{s+1}{u}_{N_z} = 0, \quad s = 0, 1, 2, \ldots. \end{aligned} \tag{3.3}$$

A realization of the iterative process requires specifying an initial distribution for function u on the zero iteration ($s = 0$). We consider two kinds of initial approximations, i.e. sinus distribution:

$$\overset{s=0}{u}_m = \sin\Big(\frac{\pi m(z - L_{sol})}{L_2 - L_1}\Big), \quad m = 1, 2, \ldots \tag{3.4}$$

and Gaussian distribution:

$$\overset{s=0}{u}_m = \exp(-(z - L_{sol})^2/a^2). \tag{3.5}$$

Parameter L_{sol} defines the coordinate with respect to which the initial distribution of EF is symmetric.

Taking into account the zero value boundary conditions, we introduce the vector $\psi = (u_1, u_2, \ldots, u_{N_z-1})$. In this case we can rewrite the equations (3.3) in matrix form:

$$\overset{s}{\Lambda}\,\overset{s+1}{\psi} = \overset{s+1}{\tilde{\lambda}}\,\overset{s+1}{\psi}. \tag{3.6}$$

$\overset{s}{\Lambda}$ is a real nonsymmetrical tridiagonal matrix:

$$\overset{s}{\Lambda} = \begin{pmatrix} -(2a_1 + b_1) & a_1 & 0 & 0 & \\ a_2 & -(2a_2 + b_2) & a_2 & 0 & \\ 0 & a_3 & -(2a_3 + b_3) & a_3 & \cdots \\ 0 & 0 & a_4 & -(2a_4 + b_4) & \\ & & \cdots & & \end{pmatrix},$$

where $a_n = -1/4\pi\Omega\varepsilon_{h,n}h^2$, $b_n = \pi\Omega\alpha_{h,n}|\overset{s}{u}_n|^2/\varepsilon_{h,n}$.

Under solution of the problem the vector $\overset{s}{\psi}$ is normalised at each iteration in accordance to the rule $\max\limits_j|\overset{s}{\psi}_j| = 1$. Iterative process completes, if the stop criterion is valid:

$$\left|\overset{s+1}{\tilde{\lambda}} - \overset{s}{\tilde{\lambda}}\right| < \delta_1|\overset{s}{\tilde{\lambda}}| + \delta_2, \quad \delta_1, \delta_2 > 0. \tag{3.7}$$

As it can be seen, the original problem of obtaining the function $u(z)$ is reduced to searching EFs and EVs of the matrix $\overset{s}{\Lambda}$. Because the matrix is nonsymmetrical, the bisection method for obtaining EVs, used in works [35, 36, 43], cannot be applied in our case. Therefore, EVs of real nonsymmetrical tridiagonal matrix are obtained using the QR-algorithm [7, 9]. Essentially, that matrix $\overset{s}{\Lambda}$ already has the Hessenberg form. This reduces the complexity of QR-algorithm to $O(N_z^3)$. However, that is an order higher than for symmetric matrix.

EVs of matrix $\overset{s}{\Lambda}$ were sorted at each iteration. After that, the maximal EV was taken for finding corresponding EF. This approach was used previously in [43] for finding soliton solutions for nonlinear Schrödinger equation in homogeneous media.

Computer simulations show that the iterative method converges for a wide range of the nonlinear parameter. Its convergence depends on a choice of initial approximation for EF. To avoid the non-convergence of iterative method we have applied the method of continuation on parameter. The soliton shape for new value of parameter is found using the soliton shape for previous value of parameter and taking into account the continuous dependence of soliton shape on this parameter. Then the convergence of iterative method was always obtained.

After finding EVs, the EF corresponding to $\overset{s+1}{\tilde{\lambda}}_m$ is calculated by inverse iteration method [7, 9, 25]. Then the obtained EF is used for constructing a new matrix $\overset{s+1}{\Lambda}$ on the following iterative step. Process is repeated, until the condition (3.7) is satisfied.

Described procedure allows to find the requiring number of EFs. However, the first EF corresponds to stable soliton, if some conditions are satisfied [43]. It should be stressed also that in general case the multiple values of EFs from the same EV can take place. However, we do not find such EVs in our experiments.

4 Computer Simulation Results

At computer simulations we restricted ourself to find the EFs with one maximum. This choice follows from our previous investigations. So, in [37, 38, 39, 40, 41, 42] we have investigated a self-formation of soliton, which locates in some layers of nonlinear 1D PC and has intensity profile similar to $ch^{-2}(z)$. In these papers we have considered the case when the optical radiation falls on PC in a direction that is perpendicular to PC layers. An appearance of soliton took place if a certain ratio between wavelength of optical radiation and thickness of layers is realised. In particular, this ratio depends on maximum intensity of laser beam.

Below we investigate the influence of problem parameters on area of soliton formation and its spatial distribution. We do not discuss the questions of formation of such solitons in practice. Nevertheless, our computer simulation results show that the soliton is stable to perturbations of its propagation direction and of its spatial distribution. Hence, one of the possible ways concludes in falling of optical radiation with soliton profile along the layers or under an angle, a little distinguishing from this direction.

During computations a left boundary of the considered domain coincides with the front point of PC (it means that L_1 is equal to L_0, Fig.1). The first layer of PC has a low dielectric permittivity and zero value of nonlinear coefficient. A coordinate L_2 of the right boundary of the domain either coincides with the right boundary of PC or is located inside of the photonic crystal layer with high dielectric permittivity. In the last case, the layer is a nonlinear one. For simplicity, let L_1 corresponds to zero value of z coordinate, and coordinate L_2 is equal to coordinate L. In other words, we consider only the domain of PC. Obviously, it does not restrict the generality.

Firstly, we discuss the influence of the maximum intensity of soliton and a ratio of optical beam frequency to the frequency of layered structure on width of soliton. Then, we consider the influence of relation between the position of maximum intensity of initial spatial distribution of EF and position of centre of layer, which is the closest to coordinate of intensity maximum, on an area of soliton formation. After that, we will discuss the jump of position of maximum intensity of soliton to one of the boundaries of nonlinear layer with changing of interaction parameters.

4.1 Soliton localization in dependence on light pulse intensity and frequency of structure.

As well as for homogeneous cubic medium, the soliton formation depends on a maximum intensity of laser radiation penetrated into PC. At fixed thickness of PC the EF becomes soliton, if the maximum intensity exceeds some critical value.

In Fig. $2a$ we show a transformation of EF with respect to increased values of $|\alpha_2|$. For small $|\alpha_2|$ ($|\alpha_2| < 4$) the EF is far from soliton shape, while for $|\alpha_2| > 5$ the soliton is formed. The width of such a soliton essentially depends on the maximum of the intensity: the larger $|\alpha_2|$, the narrower is the soliton. Nevertheless, in opposite to the case of homogeneous medium, the width of soliton, which is formed in layered structure, does not decrease always with increasing of nonlinearity. In Fig. $2a$ we can see a saturation of decreasing in soliton width. It is a consequence of layered structure with alternating linear and nonlinear layers.

The other feature of soliton formation in inhomogeneous nonlinear medium is connected to dependence of centre position for soliton profile on the presence or absence of nonlinear response in layer, which is closest to the position of the maximum intensity of a soliton (we discuss this feature below).

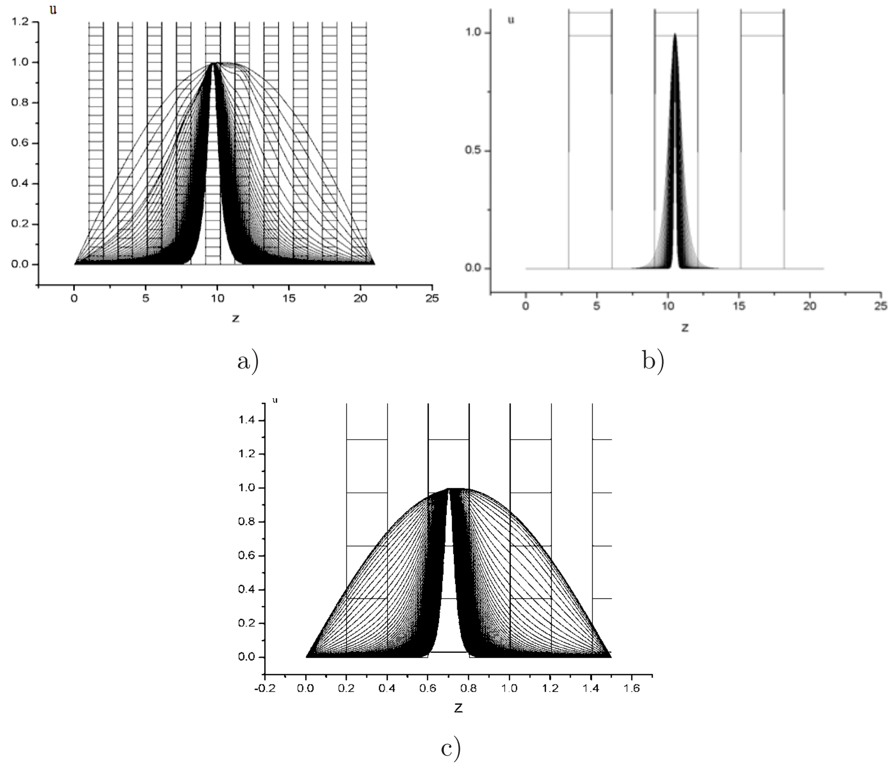

Figure 2. Dependence of EF profile on $|\alpha_2|$ (a) or on Ω (b, c) at increasing of $|\alpha_2|$ from 0 to 40 (a) and at increasing of Ω from 1 to 11 (b) or from 0.01 to 10 (c). Other parameters are the following $\Omega = 1/(4\pi)$, $\alpha_1 = 0$, $\varepsilon_1 = 1.3, \varepsilon_2 = 2.3, d_1 = d_2 = 1.0, L = 21$ (a); $\varepsilon_1 = \varepsilon_2 = 1, \alpha_1 = 0$, $\alpha_2 = -0.1$, $L = 21, d_1 = d_2 = 3$ (b), $\alpha_1 = 0, \alpha_2 = -1.0, L = 1.5, d_1 = d_2 = 0.2$ (c). A width of EF decreases with increasing of corresponding parameter. Areas with horizontal lines correspond to nonlinear layers of a photonic crystal.

The pictures Fig. 2*b,c* confirm the conclusion of our previous papers about of the opportunity for soliton formation in some layer: soliton can appear in layer if the length of the layer is greater than the wavelength of laser radiation. More explicit, a soliton appears if the wavelength of the structure is 3-4 times greater than the wavelength of laser light.

4.2 Dependence of soliton formation area on layers configuration

The location of soliton depends essentially on interposition between the centre of initial distribution for EF and the centre of nonlinear layer, which is close to it. As a rule, the maximum intensity of soliton realises in this nonlinear layer. However, if the centre of linear layer coincides with the centre of initial distribution of EF then two identical solitons may appear in the neighbouring nonlinear layers. Nevertheless, the slightest asymmetry of the layers position with respect to the centre of initial distribution of EF suppresses an appearance of such solitons and only a formation of unique soliton takes place (see, Fig. 3).

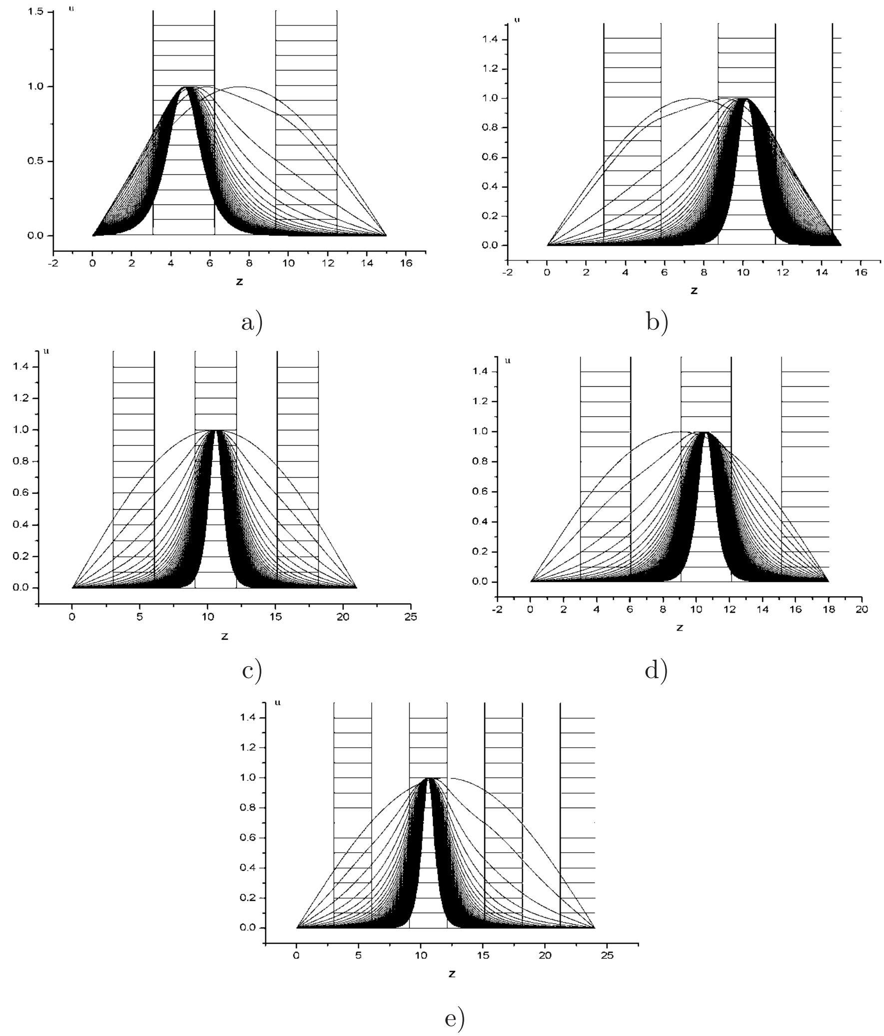

Figure 3. Transformation of EF profile in dependence on $|\alpha_2|$, which is changing from 0 to 40, for various ratio of a nonlinear layer centre position and the position of centre of initial spatial distribution of EF at $\alpha_2 = 0.0$ for $L = 15, d_1 = d_2 = 3.0$ (a) $L = 15, d_1 = d_2 = 2.9$ (b), $L = 21, d_1 = d_2 = 3.0$ (c), $L = 18, d_1 = d_2 = 3.0$ (d), $L = 24, d_1 = d_2 = 3.0$ (e) and $\alpha_1 = 0$, $\Omega = 1/(4\pi)$, $\varepsilon_1 = \varepsilon_2 = 1.0$ (homogeneous medium). A width of EF decreases with increasing of $|\alpha_2|$. Areas with horizontal lines correspond to nonlinear layers of a photonic crystal.

The unique soliton corresponds to an EV with the smallest modulus of its value. However, at the formation of twin solitons their EVs are a little different.

4.3 Jump of soliton position at changing parameters of interaction

Another significant property of soliton evolution under the gradual increasing of problem parameters is its jumping out of nonlinear layer, in which it exists

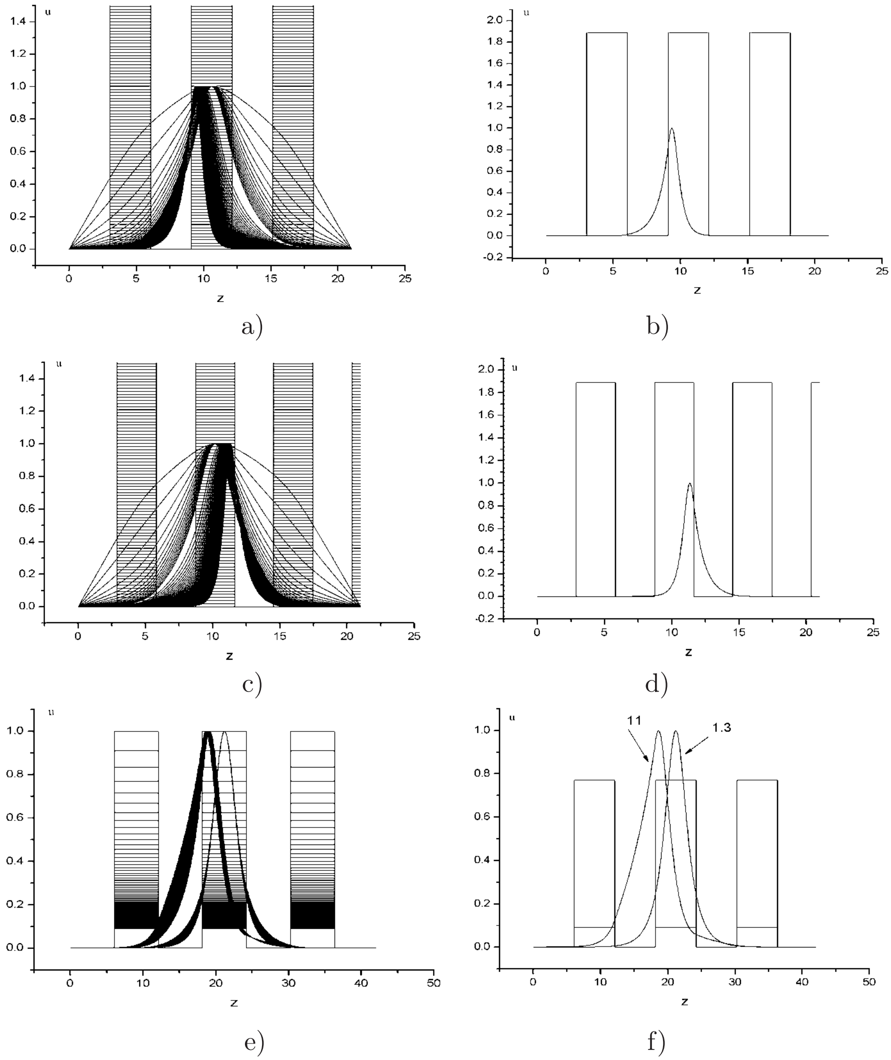

Figure 4. Jump of soliton position at increasing of $|\alpha_2|$ from 0 to 40 (a-d) for $\Omega = 1/(4\pi)$, $\varepsilon_1 = 1.69, \varepsilon_2 = 5.29, \alpha_1 = 0$, $L = 21$ at small changing of layers thickness from $d_1 = d_2 = 3.0$ (a, b) to $d_1 = d_2 = 2.9$ (c-d) or jump of soliton position at increasing of ε_2 from 1.3 to 11 (e-f) for $\Omega = 1/(4\pi)$, $\varepsilon_1 = 1, \alpha_1 = 0, \alpha_2 = -4$, $L = 42, d_1 = d_2 = 6.0$ (e-f). A width of EF decreases with increasing of $|\alpha_2|$. Areas with horizontal lines correspond to nonlinear layers of a photonic crystal. Final soliton profile for maximum value $|\alpha_2|$or ε_2 is depicted on Fig. 4b, 4d , 4f.

for values of parameters less than critical ones. The jump of soliton can be realised under various conditions. One of them corresponds to an excess of maximal light intensity of crucial value. For examples, the critical value of nonlinearity equals to $|\alpha_2| = 5.6$. Of course, this value depends on such parameters as layer thickness and diffraction coefficient. In Fig. 4*a*-*d* we see a jump of soliton location either to the left or to the right boundary of nonlinear layer in dependence of interposition between centre of layer and centre of soliton for low value of nonlinearity in PC. For considered case one half of soliton belongs to nonlinear layer and another half of soliton belongs to the linear layer. Consequently, its profile becomes asymmetrical. It is important that the greatest part of soliton can belong to linear layer.

The similar evolution of soliton location takes place at changing other parameters of problem. Fig. 4*e*-*f* illustrates the jump of soliton position at exceeding the dielectric permittivity of nonlinear layer the corresponding critical value: the larger the permittivity of nonlinear layer, the greater part of soliton displacement in a linear layer and the jump of soliton location is more pronounced. First, (for the least initial parameters values) the soliton is located in the central nonlinear layer. At variation of dielectric permittivity in the range $1 \le \varepsilon_2 \le 2.5$ the small changes of soliton profile take place. At $\varepsilon_2 = 2.6$ the soliton jumps out of the nonlinear layer and localises half in and half out of the nonlinear layer so that its maximum intensity realises at the left boundary of the layer. The further growth in dielectric permittivity ε_2 results in gradual shifting of the left part of soliton to the left boundary of PC. So, a soliton profile becomes more asymmetrical because its right part remains in the nonlinear layer without significant shifting.

Increasing the ratio between light frequency and PC frequency (parameter Ω), we get the jump of soliton location as well. In this case a soliton not only shifts to the boundary of nonlinear layer, but also its profile changes essentially. With increasing of Ω decreasing of soliton width takes place while the soliton center does not move at all. In contrast to the previous case a soliton preserves approximately symmetrical profile and its both left and right parts (out and in nonlinear layer) are shifting towards each other.

It should be stressed that the left shifts in all discussed cases are due to small non-symmetry in layers configuration: the left nonlinear layer is a little farther from the boundary then the right one. Changing of non-symmetry results in the changing of the shifts direction.

5 Conclusions

In this paper we develop the method for finding of soliton solution of Schrödinger equation with periodic linear and nonlinear properties of medium. Such structure of medium is ordinary for PC. Soliton solution is found out as EF on the base of iterative process.

This method with combination of continuation on parameter technique allows us to investigate the dependence of spatial distribution of soliton on parameters of laser pulse interaction with PC. It is very important that our algorithm allows finding the soliton in requiring domain of PC: one-layer structure or multi-layers one.

Firstly, we stress that the width of soliton essentially depends on peak intensity. This result, obviously, is similar to one for homogeneous medium with cubic nonlinear response. Nevertheless, for PC with alternating nonlinear and linear response there are some features in comparison with homogeneous nonlinear medium. In that way, decreasing or increasing maximum intensity of input laser pulse it is possible to create solitons spreading over several layers or localizing into one layer or mainly located in linear medium.

Secondly, the area of soliton localization is sensitive to the defects in thickness of PC. So, the slightest non-symmetry in layers displacement with respect to initial beam symmetry can result in soliton shift to the left or right boundary of layer.

Thirdly, soliton jump out of nonlinear layer occurs if one of the following situations is valid. The maximum intensity of optical radiation or ratio between light frequency to frequency of structure or ration between dielectric permittivity of linear and nonlinear layers exceeds some critical value under fixed other parameters.

These investigations explain the results, which were obtained by us at an investigation of soliton self-formation in some layers of PC under the propagation of laser pulse through the layered photonic structure.

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