

ON ALMOST STATISTICAL CONVERGENCE OF GENERALIZED DIFFERENCE SEQUENCES OF FUZZY NUMBERS

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Abstract. The purpose of this paper is to introduce the concepts of almost statistical convergence and strongly almost convergence of generalized difference sequences of fuzzy numbers. We obtain some results related to these concepts. It is also shown that almost Δ_χ^r -statistical convergence and strongly almost Δ_χ^r -convergence are equivalent for Δ^r -bounded sequences of fuzzy numbers.

Key words: Fuzzy number, statistical convergence, difference sequence

1. Introduction

The concept of fuzzy sets and fuzzy set operations was first introduced by Zadeh [23] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming. Matloka [15] introduced bounded and convergent sequences of fuzzy numbers and studied their some properties. Matloka [15] has shown that every convergent sequence of fuzzy numbers is bounded. Later on sequences of fuzzy numbers have been discussed by Nanda [17], Nuray and Savaş [18], Kwon [10], Savaş [20], Başarır and Mursaleen ([1, 16]) and many others.

The notion of statistical convergence was introduced by Fast [6] and Schoenberg [21], independently. Over the years and under different names statistical convergence has been discussed in the theory of fourier analysis, ergodic theory and number theory. Later on it was further investigated from the sequence space point

of view and linked with summability theory by Fridy [7], Šalát [19], Tripathy [22], Connor [2] and many others. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. Statistical convergence and its generalizations are also connected with subsets of the Stone-Čech compactification of the natural numbers. Moreover, statistical convergence is closely related to the concept of convergence in probability.

In the present paper we introduce and examine the concepts of almost statistical convergence and strongly almost convergence of generalized difference sequences of fuzzy numbers. In section 2 we give a brief information about fuzzy numbers, the operator Δ^r , the sequence $\lambda = (\lambda_n)$, statistical convergence and using the generalized difference operator Δ^r and the sequence $\lambda = (\lambda_n)$ we define the concepts of almost statistical convergence and strongly almost convergence of sequences of fuzzy numbers. In section 3 we establish some relations between strongly almost Δ_λ^r -convergence and almost Δ_λ^r -statistical convergence.

2. Definitions and Preliminaries

The definitions of statistical convergence and strong p -Cesàro convergence of a sequence of real numbers were introduced in the literature independently of one another and have followed different lines of development since their first appearance. It turns out, however, that the two definitions can be simply related to one another in general and are equivalent for bounded sequences. The idea of statistical convergence depends on the density of subsets of the set \mathbb{N} of natural numbers. The density of a subset E of \mathbb{N} is defined by

$$\delta(E) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \chi_E(k) \text{ provided the limit exists,}$$

where χ_E is the characteristic function of E . It is clear that any finite subset of \mathbb{N} has zero natural density and $\delta(E^c) = 1 - \delta(E)$.

A sequence (x_k) is said to be statistically convergent to L if for every $\varepsilon > 0$, $\delta(\{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}) = 0$. In this case we write $S - \lim x_k = L$.

Let $C(\mathbb{R}^n) = \{A \subset \mathbb{R}^n : A \text{ is compact and convex}\}$. The space $C(\mathbb{R}^n)$ has a linear structure induced by the operations $A + B = \{a + b : a \in A, b \in B\}$ and $\mu A = \{\mu a : a \in A\}$ for $A, B \in C(\mathbb{R}^n)$ and $\mu \in \mathbb{R}$. The Hausdorff distance between A and B in $C(\mathbb{R}^n)$ is defined as

$$\delta_\infty(A, B) = \max \left\{ \sup_{a \in A} \inf_{b \in B} \|a - b\|, \sup_{b \in B} \inf_{a \in A} \|a - b\| \right\}.$$

It is well known that $(C(\mathbb{R}^n), \delta_\infty)$ is a complete metric space.

A fuzzy number is a function X from \mathbb{R}^n to $[0, 1]$ which is normal, fuzzy convex, upper semi-continuous and the closure of $\{x \in \mathbb{R}^n : X(x) > 0\}$ is compact. These properties imply that for each $0 < \alpha \leq 1$, the α -level set $X^\alpha = \{x \in \mathbb{R}^n : X(x) > \alpha\}$ is a non-empty compact convex subset of \mathbb{R}^n , with support X^0 . Let $L(\mathbb{R}^n)$ denote the set of all fuzzy numbers. The linear structure of $L(\mathbb{R}^n)$ induces the addition $X + Y$ and the scalar multiplication μX , $\mu \in \mathbb{R}$, in terms of α -level sets, by

$$[X + Y]^\alpha = X^\alpha + Y^\alpha, \quad [\mu X]^\alpha = \mu X^\alpha$$

for each $0 < \alpha \leq 1$.

Define, for each $1 \leq q < \infty$,

$$d_q(X, Y) = \left(\int_0^1 \delta_\infty(X^\alpha, Y^\alpha)^q d\alpha \right)^{1/q}$$

and $d_\infty(X, Y) = \sup_{0 \leq \alpha \leq 1} \delta_\infty(X^\alpha, Y^\alpha)$. Clearly $d_\infty(X, Y) = \lim_{q \rightarrow \infty} d_q(X, Y)$ with $d_q(X, Y) \leq d_s(X, Y)$ if $q \leq s$ [3].

Throughout the paper, d will denote d_q with $1 \leq q \leq \infty$.

The famous space \hat{c} of all almost convergent sequences was introduced by Lorentz [12] and several authors such as Duran [4], King [8] have studied almost convergent sequences. Maddox ([13],[14]) has defined x to be strongly almost convergent to a number L if

$$\lim_n \frac{1}{n} \sum_{k=1}^n |x_{k+m} - L| = 0, \quad \text{uniformly in } m.$$

The idea of difference sequences for real numbers was first introduced by Kızmaz [9] and this concept was generalized by Et and Başarır [5].

Let w be the set of all sequences of fuzzy numbers. The operator $\Delta^r : w \rightarrow w$ is defined by

$$(\Delta^0 X)_k = X_k, \quad (\Delta^1 X)_k = \Delta^1 X_k = X_k - X_{k+1}, \quad (k = 0, 1, \dots), \quad \Delta^r = \Delta^1 \circ \Delta^{r-1}, \quad (r \geq 2).$$

The generalized de la Vallée-Pousin mean is defined by

$$t_n(x) = \frac{1}{\lambda_n} \sum_{k \in I_n} x_k,$$

where $\lambda = (\lambda_n)$ is a non-decreasing sequence of positive numbers such that $\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$, $\lambda_n \rightarrow \infty$ as $n \rightarrow \infty$ and $I_n = [n - \lambda_n + 1, n]$.

A sequence $x = (x_k)$ is said to be (V, λ) -summable to a number L [11] if $t_n(x) \rightarrow L$ as $n \rightarrow \infty$. (V, λ) -summability reduces to $(C, 1)$ summability when $\lambda_n = n$ for all $n \in \mathbb{N}$.

Now we will extend the notions of strongly almost convergence and almost statistical convergence of sequences of real numbers to the idea of sequences of fuzzy numbers using the generalized difference operator Δ^r and the sequence $\lambda = (\lambda_n)$.

DEFINITION 1. Let $X = (X_k)$ be a sequence of fuzzy numbers. Then the sequence $X = (X_k)$ of fuzzy numbers is said to be almost Δ_λ^r -statistically convergent to the fuzzy number X_0 if for every $\varepsilon > 0$,

$$\lim_{n \rightarrow \infty} \lambda_n^{-1} |\{k \in I_n : d(\Delta^r X_{k+i}, X_0) \geq \varepsilon\}| = 0, \quad \text{uniformly in } i.$$

In this case we write $X_k \rightarrow X_0 \left(\hat{S}(\Delta_\lambda^r) \right)$ or $\hat{S}(\Delta_\lambda^r) - \lim X_k = X_0$.

The set of all almost Δ_λ^r -statistically convergent sequences of fuzzy numbers is denoted by $\hat{S}(\Delta_\lambda^r)$. In the special case $\lambda_n = n$ for all $n \in \mathbb{N}$, we shall write $\hat{S}(\Delta^r)$ instead of $\hat{S}(\Delta_\lambda^r)$ and we said that X is almost Δ^r -statistically convergent to the fuzzy number X_0 .

DEFINITION 2. Let $X = (X_k)$ be a sequence of fuzzy numbers and $p = (p_k)$ be a sequence of strictly positive real numbers. Then the sequence $X = (X_k)$ is said to be strongly almost Δ_λ^r -convergent if there is a fuzzy number X_0 such that

$$\lim_{n \rightarrow \infty} \lambda_n^{-1} \sum_{k \in I_n} [d(\Delta^r X_{k+i}, X_0)]^{p_k} = 0, \text{ uniformly in } i.$$

In this case we write $X_k \rightarrow X_0$ $([w, p, \Delta_\lambda^r])$. We shall use $[w, p, \Delta_\lambda^r]$ to denote the set of all strongly almost Δ_λ^r -convergent sequences of fuzzy numbers. In the special case $\lambda_n = n$ for all $n \in \mathbb{N}$, and $p_k = 1$ for all $k \in \mathbb{N}$, we shall write $[w, p, \Delta^r]$ and $[w, \Delta_\lambda^r]$, respectively, instead of $[w, p, \Delta_\lambda^r]$. If $X \in [w, p, \Delta^r]$ then we say that X is strongly almost Δ^r -Cesàro summable.

DEFINITION 3. Let $X = (X_k)$ be a sequence of fuzzy numbers. Then the sequence $X = (X_k)$ of fuzzy numbers is said to be Δ^r -bounded if the set $\{\Delta^r X_k : k \in \mathbb{N}\}$ of fuzzy numbers is bounded. By $\ell_\infty(\Delta^r)$ we shall denote the set of all Δ^r -bounded sequences of fuzzy numbers.

3. Main Results

In this section we give some inclusion relations between strongly almost Δ_λ^r -convergence and almost Δ_λ^r -statistical convergence and show that they are equivalent for Δ^r -bounded sequences of fuzzy numbers. We also study the inclusion $\hat{S}(\Delta^r) \subset \hat{S}(\Delta_\lambda^r)$ under certain restrictions on $\lambda = (\lambda_n)$.

Theorem 1. *If $(X_k), (Y_k) \in \hat{S}(\Delta_\lambda^r)$ and $c \in \mathbb{R}$ then*

- i) $\hat{S}(\Delta_\lambda^r) - \lim cX_k = c\hat{S}(\Delta_\lambda^r) - \lim X_k$
- ii) $\hat{S}(\Delta_\lambda^r) - \lim (X_k + Y_k) = \hat{S}(\Delta_\lambda^r) - \lim X_k + \hat{S}(\Delta_\lambda^r) - \lim Y_k$

Proof.

- i) Let $\hat{S}(\Delta_\lambda^r) - \lim X_k = X_0$, $c \in \mathbb{R}$ and $\varepsilon > 0$ be given. Then the proof follows from the following inequality:

$$\begin{aligned} & \lambda_n^{-1} |\{k \in I_n : d(\Delta^r cX_{k+i}, cX_0) \geq \varepsilon\}| \\ & \leq \lambda_n^{-1} \left| \left\{ k \in I_n : d(\Delta^r X_{k+i}, X_0) \geq \frac{\varepsilon}{|c|} \right\} \right|. \end{aligned}$$

- ii) Suppose that $\hat{S}(\Delta_\lambda^r) - \lim X_k = X_0$ and $\hat{S}(\Delta_\lambda^r) - \lim Y_k = Y_0$. By Minkowski's inequality we get

$$d(\Delta^r X_{k+i} + \Delta^r Y_{k+i}, X_0 + Y_0) \leq d(\Delta^r X_{k+i}, X_0) + d(\Delta^r Y_{k+i}, Y_0).$$

Therefore given $\varepsilon > 0$ we have

$$\begin{aligned} & \lambda_n^{-1} |\{k \in I_n : d(\Delta^r X_{k+i} + \Delta^r Y_{k+i}, X_0 + Y_0) \geq \varepsilon\}| \\ & \leq \lambda_n^{-1} \left| \left\{ k \in I_n : d(\Delta^r X_{k+i}, X_0) \geq \frac{\varepsilon}{2} \right\} \right| + \lambda_n^{-1} \left| \left\{ k \in I_n : d(\Delta^r Y_{k+i}, Y_0) \geq \frac{\varepsilon}{2} \right\} \right|. \end{aligned}$$

Hence $\hat{S}(\Delta_\lambda^r) - \lim (X_k + Y_k) = X_0 + Y_0$. ■

The following theorem shows that almost Δ_λ^r -statistical convergence and strongly almost Δ_λ^r -convergence are equivalent for Δ^r -bounded sequences of fuzzy numbers.

Theorem 2. *Let the sequence (p_k) be bounded and $X = (X_k)$ be a sequence of fuzzy numbers. Then*

- i) $X_k \rightarrow X_0 ([w, p, \Delta_\lambda^r])$ implies $X_k \rightarrow X_0 (\hat{S}(\Delta_\lambda^r))$,
- ii) $X \in \ell_\infty(\Delta^r)$ and $X_k \rightarrow X_0 (\hat{S}(\Delta_\lambda^r))$ imply $X_k \rightarrow X_0 ([w, p, \Delta_\lambda^r])$,
- iii) $\hat{S}(\Delta_\lambda^r) \cap \ell_\infty(\Delta^r) = [w, p, \Delta_\lambda^r] \cap \ell_\infty(\Delta^r)$.

Proof.

i) Let $\varepsilon > 0$ and $X_k \rightarrow X_0 ([w, p, \Delta_\lambda^r])$. Then we can write

$$\begin{aligned} \frac{1}{\lambda_n} \sum_{k \in I_n} [d(\Delta^r X_{k+i}, X_0)]^{p_k} &\geq \frac{1}{\lambda_n} \sum_{\substack{k \in I_n \\ d(\Delta^r X_{k+i}, X_0) \geq \varepsilon}} [d(\Delta^r X_{k+i}, X_0)]^{p_k} \\ &\geq \frac{1}{\lambda_n} \sum_{\substack{k \in I_n \\ d(\Delta^r X_{k+i}, X_0) \geq \varepsilon}} \varepsilon^{p_k} \geq \frac{1}{\lambda_n} \sum_{\substack{k \in I_n \\ d(\Delta^r X_{k+i}, X_0) \geq \varepsilon}} \min(\varepsilon^h, \varepsilon^H) \\ &\geq \frac{1}{\lambda_n} |\{k \in I_n : d(\Delta^r X_{k+i}, X_0) \geq \varepsilon\}| \min[\varepsilon^h, \varepsilon^H], \end{aligned}$$

where $0 < h = \inf p_k \leq p_k \leq \sup p_k = H < \infty$. Hence $X_k \rightarrow X_0 (\hat{S}(\Delta_\lambda^r))$.

ii) Suppose that $X \in \ell_\infty(\Delta^r)$ and $X_k \rightarrow X_0 (\hat{S}(\Delta_\lambda^r))$. Since $X \in \ell_\infty(\Delta^r)$, there is a constant $T > 0$ such that $d(\Delta^r X_{k+i}, X_0) \leq T$. Given $\varepsilon > 0$, we have

$$\begin{aligned} \frac{1}{\lambda_n} \sum_{k \in I_n} [d(\Delta^r X_{k+i}, X_0)]^{p_k} &= \frac{1}{\lambda_n} \sum_{\substack{k \in I_n \\ d(\Delta^r X_{k+i}, X_0) \geq \varepsilon}} [d(\Delta^r X_{k+i}, X_0)]^{p_k} \\ &\quad + \frac{1}{\lambda_n} \sum_{\substack{k \in I_n \\ d(\Delta^r X_{k+i}, X_0) < \varepsilon}} [d(\Delta^r X_{k+i}, X_0)]^{p_k} \\ &\leq \frac{1}{\lambda_n} \sum_{\substack{k \in I_n \\ d(\Delta^r X_{k+i}, X_0) \geq \varepsilon}} \max(T^h, T^H) + \frac{1}{\lambda_n} \sum_{\substack{k \in I_n \\ d(\Delta^r X_{k+i}, X_0) < \varepsilon}} \varepsilon^{p_k} \\ &\leq \max(T^h, T^H) \frac{1}{\lambda_n} |\{k \in I_n : d(\Delta^r X_{k+i}, X_0) \geq \varepsilon\}| + \max(\varepsilon^h, \varepsilon^H). \end{aligned}$$

Hence $X_0 \in [w, p, \Delta_\lambda^r]$.

iii) Follows from (i) and (ii).

■

Theorem 3. If a sequence $X = (X_k)$ is almost Δ^r -statistically convergent to the fuzzy number X_0 and $\liminf_n (\lambda_n/n) > 0$, then it is almost Δ_λ^r -statistically convergent to X_0 .

Proof. Given $\varepsilon > 0$ we have

$$|\{k \leq n : d(\Delta^r X_{k+i}, X_0) \geq \varepsilon\}| \supseteq |\{k \in I_n : d(\Delta^r X_{k+i}, X_0) \geq \varepsilon\}|.$$

Therefore,

$$\begin{aligned} \frac{1}{n} |\{k \leq n : d(\Delta^r X_{k+i}, X_0) \geq \varepsilon\}| &\geq \frac{1}{n} |\{k \in I_n : d(\Delta^r X_{k+i}, X_0) \geq \varepsilon\}| \\ &\geq \frac{\lambda_n}{n} \cdot \frac{1}{\lambda_n} |\{k \in I_n : d(\Delta^r X_{k+i}, X_0) \geq \varepsilon\}|. \end{aligned}$$

Taking limit as $n \rightarrow \infty$ and using $\liminf_n (\lambda_n/n) > 0$, we get that X is Δ_λ^r -statistically convergent to X_0 . ■

Theorem 4. Let $0 < p_k \leq q_k$ and (q_k/p_k) be bounded. Then $[w, q, \Delta_\lambda^r] \subset [w, p, \Delta_\lambda^r]$.

Proof. Let $X \in [w, q, \Delta_\lambda^r]$. Write $w_{k,i} = [d(\Delta^r X_{k+i}, X_0)]^{q_k}$ and $\mu_k = \frac{p_k}{q_k}$ so that $0 < \mu < \mu_k \leq 1$ for each k .

We define the sequences $(u_{k,i})$ and $(v_{k,i})$ as follows:

Let $u_{k,i} = w_{k,i}$ and $v_{k,i} = 0$ if $w_{k,i} \geq 1$, and let $u_{k,i} = 0$ and $v_{k,i} = w_{k,i}$ if $w_{k,i} < 1$. Then it is clear that for all $k \in \mathbb{N}$ we have $w_{k,i} = u_{k,i} + v_{k,i}$, $w_{k,i}^{\mu_k} = u_{k,i}^{\mu_k} + v_{k,i}^{\mu_k}$. Now it follows that $u_{k,i}^{\mu_k} \leq u_{k,i} \leq w_{k,i}$ and $v_{k,i}^{\mu_k} \leq v_{k,i}$. Therefore

$$\begin{aligned} \lambda_n^{-1} \sum_{k \in I_n} w_{k,i}^{\mu_k} &= \lambda_n^{-1} \sum_{k \in I_n} (u_{k,i}^{\mu_k} + v_{k,i}^{\mu_k}) \\ &\leq \lambda_n^{-1} \sum_{k \in I_n} w_{k,i} + \lambda_n^{-1} \sum_{k \in I_n} v_{k,i}^{\mu_k}. \end{aligned}$$

Since $\mu < 1$, for each n we have

$$\begin{aligned} \lambda_n^{-1} \sum_{k \in I_n} v_{k,i}^{\mu_k} &= \sum_{k \in I_n} (\lambda_n^{-1} v_{k,i})^\mu (\lambda_n^{-1})^{1-\mu} \\ &\leq \left(\sum_{k \in I_n} [(\lambda_n^{-1} v_{k,i})^\mu]^{\frac{1}{\mu}} \right)^\mu \left(\sum_{k \in I_n} [(\lambda_n^{-1})^{1-\mu}]^{\frac{1}{1-\mu}} \right)^{1-\mu} \\ &= \left(\lambda_n^{-1} \sum_{k \in I_n} v_{k,i} \right)^\mu \end{aligned}$$

by Hölder's inequality, and thus

$$\lambda_n^{-1} \sum_{k \in I_n} w_{k,i}^{\mu_k} \leq \lambda_n^{-1} \sum_{k \in I_n} w_{k,i} + \left(\lambda_n^{-1} \sum_{k \in I_n} v_{k,i} \right)^{\mu}.$$

Hence $X \in [w, \Delta_{\lambda}^r]$. ■

Theorem 5. $[w, \Delta_{\lambda}^r]_{\infty} = \ell_{\infty}(\Delta^r)$, where

$$[w, \Delta_{\lambda}^r]_{\infty} = \left\{ X = (X_k) : \sup_{n,i} \lambda_n^{-1} \sum_{k \in I_n} [d(\Delta^r X_{k+i}, \bar{0})] < \infty \right\}.$$

Proof. Let $X \in [w, \Delta_{\lambda}^r]_{\infty}$. Then there exists a constant $K_1 > 0$ such that

$$\frac{1}{\lambda_1} d(\Delta^r X_{1+i}, \bar{0}) \leq \sup_{n,i} \frac{1}{\lambda_n} \sum_{k \in I_n} d(\Delta^r X_{k+i}, \bar{0}) \leq K_1 \text{ for all } i$$

and so we have $X \in \ell_{\infty}(\Delta^r)$.

Conversely, let $X \in \ell_{\infty}(\Delta^r)$. Then there exists a constant $K_2 > 0$ such that $d(\Delta^r X_j, \bar{0}) \leq K_2$ for all j , and so

$$\frac{1}{\lambda_n} \sum_{k \in I_n} d(\Delta^r X_{k+i}, \bar{0}) \leq \frac{K_2}{\lambda_n} \sum_{k \in I_n} 1 \leq K_2 \text{ for all } k \text{ and } i.$$

Thus $X \in [w, \Delta_{\lambda}^r]_{\infty}$. ■

4. Conclusion

Giving particular values to the sequence $\lambda = (\lambda_n)$, p and r we obtain some sequence spaces which are the special cases of the sequence spaces that we have defined. The most of the results proved in the previous sections will be true for these spaces.

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Apie beveik statistinį apibendrintų skirtuminių fuzzy skaičių sekų konvergavimą

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Šio straipsnio tikslas supažindinti su beveik statistinio ir stipriai beveik statistinio apibendrintų fuzzy skaičių konvergavimo sąvokomis. Straipsnyje taip pat parodyta, kad beveik Δ_λ^r – statistinis konvergavimas ir stipriai beveik Δ_λ^r – statistinis konvergavimas yra ekvivalentūs Δ^r – apibendrintoms fuzzy skaičių sekoms.