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A MODEL OF INTENSIVE OIL BURNOUT FROM GLASS FABRIC

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ABSTRACT

This paper continues the previous investigations [1]-[3] for process of intensive oil burnout in glass fabric.

1. INTRODUCTION

Several years ago the mathematical simulation group of Institute of Mathematics of Latvian Academy of Sciences and University of Latvia, in cooperation with the Valmiera Glass Plant, made a start to the modelling of the process of burning oil out from the glass fabric.

The glass fabric is produced from extremely small fibers forming 'threads' from which the fabric is to be woven. During the initial stages of the technological process - manufacturing the threads and weaving the fabric - the glass is covered with an oil film. Once the weaving is complete, the oil must be burned away, so that the fabric can serve for insulation, fire-protective and other purposes. The burnout is performed in a special furnace through which the fabric is pulled. At the top and bottom of the furnace, in parallel with the fabric sheet plane there are placed steel plates heated by diesel/residual-oil fuel or gas. Therefore the fabric after it has entered the furnace is heated extremely fast; the oil ignites and burns out from the fabric, which results in whitening the latter. The whole process in the furnace last a few seconds (typically, 3-4 s). In turn, having left the furnace, the fabric cools down rapidly, which may result in impermissible degradation of its mechanical strength. Our challenge is therefore to define, by means of mathematical modelling, the influence of different factors (such as the pull-through speed of the fabric, the properties of the metallic plates, the fabric thickness, etc.) on the fabric. In the above mentioned works [1]-[3] it was estimated that, to describe adequately the technological process by means of a mathematical model based on Stefan-Boltzmann's law of the radiant heat transfer, we should take into account the reflected radiation between the fabric surface and the heaters. This means that any point on the fabric surface receives heat from all the points on the heater plate (as well as reflects it to these points). In other words, the boundary conditions of Stefan-Boltzmann's type should be written in the integral form. It might be noted that in the mentioned publications a process was modelled in which the heater placed above the fabric had a lower temperature; this means that the effect of reflected radiation between the top of the fabric and the top heater could be discounted, with the classic Stefan-Boltzmann boundary condition employed instead.

As concerns thicker fabrics, to burn away all the oil it was necessary to rise the temperature of the top heater. In this case, the effect of reflected radiation between the fabric and both heaters should be accounted for.

In this work a mathematical model is given that describes this more intensive technological process.

2. DESCRIPTION OF THE TECHNOLOGICAL PROCESS AND ITS MATHEMATICAL MODEL

The sizes of the active zone situated in the furnace in which the fabric is heated are as follows: length L, width D_0 and height (the distance between heating plates) H. The fabric of width D and thickness δ is pulled through the furnace with velocity v. Since in a real technological process the length of the fabric ($L \approx D$) and each of the heaters possesses its own (though constant along the length and width of the furnace) temperature, we may assume (at least in the first stage of the modelling) that the temperature changes only along the fabric movement and perpendicularly to the fabric plane, we may present the differential equation for the temperature field $T(x, y, t)$ in the fabrics (see [1])

$$\rho c_p\left(\frac{\partial \tilde{T}}{\partial t} + v\frac{\partial \tilde{T}}{\partial x}\right) = \frac{\partial}{\partial x}\left(k\frac{\partial \tilde{T}}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial \tilde{T}}{\partial y}\right) + \tilde{R}_1, \tag{2.1}$$

$$0 < x < L,\ 0 < y < \delta,\ t > 0,$$

where ρ, c_p and k are respectively the density, specific heat and thermal conductivity of the glass fabric, but x-axis is oriented along the fabric's movement and y-axis is perpendicular to the fabric. In turn, the term $\tilde{R}_1$ refers to the process of oil burnout (this term will be specified further).

The fabric thickness δ is relatively small as compared with other geometrical parameters; we therefore will define the temperature averaged over the fabric thickness

$$T(x,t) = \frac{1}{\delta}\int\limits_0^\delta \tilde{T}(x,y,t)dy. \tag{2.2}$$

Integrating equation (2.1), we obtain

$$\rho c_p(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x}) = \frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + \frac{1}{\delta}\left[k\frac{\partial \tilde{T}}{\partial y}\Big|_{y=\delta} - k\frac{\partial \tilde{T}}{\partial y}\Big|_{y=0}\right] + R_1, \tag{2.3}$$

$$0 < x < L,\ t > 0.$$

We shall now describe the heat transfer proceeding between the fabric surface and the surroundings. Here we will consider two mechanisms:

1) radiative heat exchange with the heaters according to Stefan-Bortzmann's law;

2) convective heat exchange with the gas whose temperature in the furnace is T_g.

In the literature, various techniques are proposed for defining the heat transfer coefficient $\alpha(T)$. We used, after having performed a series of numerical experiments, a relatively simple expression from [4], because it became evident that the main role here was played by the radiative heat transfer:

$$\alpha(T) = Nu\frac{k_g}{L},\ Nu = 0.044Re^{0.77}\frac{T}{T_g},\ Re = \frac{LU_g}{\nu_g},$$

where k_g, ν_g and U_g are relatively the thermal conductivity, kinematic viscosity and velocity of the furnace gas. We assume that the gas velocity coincides with the speed of fabric movement: $U_g = v$. Then the boundary condition at the top surface of fabric $y = \delta$, will be:

$$k\frac{\partial \tilde{T}}{\partial y} = \varepsilon_f\sigma[T_{ht}^4 - \tilde{T}^4] + \alpha(\tilde{T})(T_g - \tilde{T}), \tag{2.4}$$

and at the bottom one, $y = 0$, correspondingly:

$$-k\frac{\partial \tilde{T}}{\partial y} = \varepsilon_f\sigma[T_{hb}^4 - \tilde{T}^4] + \alpha(\tilde{T})(T_g - \tilde{T}). \tag{2.5}$$

Here ε_f is the fabric emissivity, σ is the Stefan-Boltzmann constant, $T_{ht}(T_{hb})$ is the temperature of the top (bottom) heater. After the very first numerical experiments it became evident ([1], [3]) (even in the absence of the burnout process impact, that is, when $R_1 = 0$ in equation (2.3)) that the reciprocal reflection of the heat flow between the fabric and the bottom heating plate (which is nearer to the fabric than the top plate and possesses a higher temperature) must be taken into account. Then the boundary condition (2.5) on the fabric bottom surface transforms into the following set of equations (see [1] – [3], with the general theory given in [5], chap.8):

$$-k\frac{\partial\tilde{T}}{\partial y} = \frac{\varepsilon_f}{1-\varepsilon_f}[J_f - \sigma\tilde{T}^4] + \alpha(\tilde{T})(T_g - \tilde{T}), \tag{2.6}$$

$$\times J_f(x,t) = \varepsilon_f\sigma\tilde{T}^4(x,0,t) + (1-\varepsilon_f)$$

$$\int_0^L J_h(\xi,t)\frac{a^2}{2[(\xi-x)^2+a^2]^{3/2}}d\xi, \tag{2.7}$$

$$J_h(x,t) = \varepsilon_h\sigma T_{hb}^4 + (1-\varepsilon_h)\int_0^L J_f(\xi,t)\frac{a^2}{2[(\xi-x)^2+a^2]^{3/2}}d\xi, \tag{2.8}$$

where ε_h is the emissivity of the bottom heater, $a = a_b$ is the distance between the bottom edge of the fabric and the bottom heater.

We are coming now to the description of the burnout process. Assuming that the oil burns via a simple one-step Arrhenius reaction, we obtain:

$$R_1 = c(x,t)\Delta HA\,exp(-\frac{E_a}{RT}),$$

where ΔH is the heat of reaction of the oil combustion, A - pre-exponential Arrhenius constant, E_a - activation energy of oil, R -gas constant.

In turn, $c(x,y)$ is the non-burnt oil concentration varying as described by the differential equation

$$\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial x} = -cA\,exp(-\frac{E_a}{RT}).$$

For a real technological process in non-stop production the dependence on time t is lost. Further, taking into consideration that the fabric thickness δ is small, we assume that the temperature variations along the fabric thickness can be discounted. Then from relationship (2.2) we derive for a non-stationary process

$$\tilde{T}(x,0,t) = \tilde{T}(x,\delta,t) = T(x,t),$$

and for a stationary process

$$\tilde{T}(x,0) = \tilde{T}(x,\delta) = T(x).$$

At last, estimation of various factors of the process shows (see [1]) that the heat conduction in the x-direction is of little importance as compared with other factors. This allows us to present the following mathematical model:

$$c_p\rho v\frac{dT}{dx} = \frac{\varepsilon_f}{\delta}\left[\sigma(T_{ht}^4 - T^4) + \frac{J_f - \sigma T^4}{1-\varepsilon_f}\right]$$
$$+\frac{2\alpha(T)}{\delta}(T_g - T) + c\Delta H A\, exp(-\frac{E_a}{RT}), \qquad (2.9)$$

$$v\frac{dc}{dx} = -cA\exp(-\frac{E_a}{RT}), \qquad (2.10)$$

$$J_f(x) = \varepsilon_f\sigma T^4(x) + (1-\varepsilon_f)\int\limits_0^L J_h(\xi)\frac{a^2 d\xi}{2[(\xi-x)^2+a^2]^{3/2}}, \qquad (2.11)$$

$$J_h(x) = \varepsilon_h\sigma T_{hb}^4 + (1-\varepsilon_h)\int\limits_0^L J_f(\xi)\frac{a^2 d\xi}{2[(\xi-x)^2+a^2]^{3/2}}, \qquad (2.12)$$

$$T(0) = T^0, \ \ c(0) = c^0. \qquad (2.13)$$

Here $T^0(c^0)$ is the temperature of fabric at the furnace entrance (the initial concentration of oil in the fabric).

3. A MATHEMATICAL MODEL FOR THE INTENSIVE OIL BURNOUT

Glas fabric are used for various purposes, as decorative clothes, electric or thermal insulation material (the latter may be used, for example, for fire-protective clothes). Accordingly, the thicknesses of such fabrics may be different. For a thicker fabric it is necessary, with the aim of attaining the complete oil burnout, to rise the temperature also on the top heating plate. For this case, one should take into account the heat reflection between the top surface of fabric and the top heater. For this, the set of equations (2.9)-(2.13) is to be substituted for a more generalized one. For a general non-stationary process this will read as follows:

$$c_p\rho(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x}) = \frac{\varepsilon_f}{\delta(1-\varepsilon_f)}(J_{ft} + J_{fb} - 2\sigma T^4)$$
$$+\frac{2\alpha(T)}{\delta}(T_g - T) + c\Delta H A\, exp(-\frac{E_a}{RT}), \qquad (3.1)$$

$$\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial x} = -cA\, exp(-\frac{E_a}{RT}), \qquad (3.2)$$

$$J_{ft}(x,t) = \varepsilon_f\sigma T^4 + (1-\varepsilon_f)\int\limits_0^L J_{ht}(\xi,t)\frac{a_t^2 d\xi}{2[(\xi-x)^2+a_t^2]^{3/2}}, \qquad (3.3)$$

$$J_{ht}(x,t) = \varepsilon_h\sigma T_{ht}^4 + (1-\varepsilon_h)\int\limits_0^L J_{ft}(\xi,t)\frac{a_t^2 d\xi}{2[(\xi-x)^2+a_t^2]^{3/2}}, \qquad (3.4)$$

$$J_{fb}(x,t) = \varepsilon_f \sigma T^4 + (1-\varepsilon_f)\int\limits_0^L J_{hb}(\xi,t)\frac{a^2 d\xi}{2[(\xi-x)^2+a^2]^{3/2}}, \quad (3.5)$$

$$J_{hb}(x,t) = \varepsilon_h \sigma T_{hb}^4 + (1-\varepsilon_h)\int\limits_0^L J_{fb}(\xi,t)\frac{a^2 d\xi}{2[(\xi-x)^2+a^2]^{3/2}}, \quad (3.6)$$

$$T\Big|_{x=0} = T^0(t), \quad c\Big|_{x=0} = c^0(t), \quad (3.7)$$

$$T\Big|_{t=0} = T_0^0(x), \quad c\Big|_{t=0} = c_0^0(x). \quad (3.8)$$

Here $a_t = H - \delta - a$, with index $t(b)$ relating to the top (bottom) part of the fabric and top (bottom) heater plate.

Normally, the oil concentration distribution in the fabric at the initial time moment $c_0^0(x)$ coincides with the oil concentration at the furnace entrance $c^0(t)$ and is constant, i.e. $c^0(t) = c_0^0(x) \equiv c^0$.

As follows from comparison of boundary conditions (2.5) and (2.6) – (2.8), both the models – taking and not taking the heat flow reflection into account – may be described by a single basic equation (3.1) if the flow J_f in the non-reflection model is defined as follows:

$$J_f = (1-\varepsilon_f)\sigma T_h^4 + \varepsilon_f \sigma T^4. \quad (3.9)$$

4. THE SOLUTION OF THE MATHEMATICAL MODEL

For a real industrial process constant conditions are needed. This means that we have to describe a stationary process: $\frac{\partial T}{\partial t} = 0$, $\frac{\partial c}{\partial t} = 0$. It should be noted that in the case of a non-stationary process the algorithm is essentially the same. Further we will consider a generalized process of intensive treatment (3.1) – (3.8). The calculations performed earlier ([1]-[3])show that at some local points (within the burning zone) the temperature varies very fast. Therefore for our numerical computations a piecewise-uniform grid (with a smaller step in the burning zone) was used. We have exploited an iterative algorithm, considering a process with boundary conditions (2.4), (2.5) as a first approximation; this means that at the first step we find the temperature distribution for the process with no reflection. Secondly, in each subinterval $[x_n, x_{n+1}]$ we consider fluxes (3.3) – (3.6) and similarly to publication [1] we can derive (for example, from (3.3)) the following:

$$\begin{aligned} J_{ft}(x_n) &= \varepsilon_f \sigma T^4(x_n) + (1-\varepsilon_f)\sum_{j=0}^{N-1}\frac{J_{ht}(x_j)+J_{ht}(x_{j+1})}{4} \times \\ &\left[\frac{x_{j+1}-x_n}{\sqrt{(x_{j+1}-x_n)^2+a_t^2}} - \frac{x_j - x_n}{\sqrt{(x_j-x_n)^2+a_t^2}}\right]. \end{aligned}$$

Table 1.

$\tilde{x}$	$\tilde{T}_1$	$\tilde{T}_2$	$\tilde{T}_3$	$\tilde{T}_4$
0.16	0.67	0.58	0.78	0.65
0.19	0.72	0.66	0.86	0.77
0.21	0.76	0.73	1.21	1.32
0.24	0.81	0.79	1.13	1.12
0.26	0.87	0.84	1.10	1.09
0.27	0.92	0.89	1.08	1.07
0.28	1.20	1.27	1.07	1.06
0.29	1.16	1.18	1.05	1.05
0.30	1.13	1.13	1.03	1.04

Expression of the same kind might be written for the other fluxes (3.4) – (3.6). All the fluxes $J_{ft}(x_n)$, $J_{fb}(x_n)$, $n = 0, 1, \ldots, N$, we can approximate with the classical cubic splines (with the continuous second derivative at the grid points x_n) and substitute these expressions into equation (3.1). The set of ordinary differential equations (3.1), (3.2) is computed based on the Maple package using the dsolve program (with the lsode – implicit Adams method). If for flux J_{ft} (or for J_{ft} and J_{fb}) formula (3.9) is employed, it is obvious that this computation stage becomes simpler.

Comparative computations have been performed for the following variants of the stationary process: reflected radiation is not taken in to account, that is, flux J is computed by formula (3.9); the top heater is cooler, so the model (2.9) – (2.13) can be exploited; there is an intensive burnout, therefore we use the "full" system (3.1) – (3.8).

For numerical calculation we use following date: $L = 1.166$m, $\delta = 0.2$mm, $a = 0.15$m, $a_t = 0.20$m, $\varepsilon_f = 0.92$, $\varepsilon_h = 0.8$, $\rho = 1100$kg$/m^3$, $c_p = 690.82$J/kg/K, $\sigma = 5.6703 \times 10^{-8}W/m^2/K^4$, $E_a = 160$kJ/mol, $v = 0.33$m/sek, $\Delta H = 1.207 \times 10^7$J/mol, $A = 1.0 \times 10^9$/sek, $R = 8.31441$J/K/mol.

Temperature distribution in the fabric is represented in the following table 1, where
$\tilde{T}_1(x) = T(x)/T_{hb}$ with T_{hb}=1123 K, T_{ht}=973 K and without the reflection;
$\tilde{T}_2(x) = T(x)/T_{hb}$ with T_{hb}=1123 K, T_{ht}=973 K and with the reflection;
$\tilde{T}_3(x) = T(x)/T_{hb}$ with $T_{hb} = T_{ht}$=1123 K and without the reflection;
$\tilde{T}_4(x) = T(x)/T_{hb}$ with $T_{hb} = T_{ht}$=1123 K and with the reflection.

One can see that in the case of intensive burnout the maximum temperature in the burning zone increases and the burning process itself starts essentially nearer to the entrance of the furnace.

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ALIEJAUS IŠDEGINIMO IŠ STIKLO GAMINIŲ MATEMATINIS MODELIS

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Darbe toliau sistemingai nagrinėjami stiklo gaminimo procesų matematiniai modeliai. Tiriama įvairių faktorių (stiklo masės judėjimo greičio, metalo plokštelių savybių, gaminio storio) įtaka stiklo gaminio kokybei. Kadangi aliejaus išdeginimas vyksta labai aukštoje temperatūroje ir labai trumpai, tai matematiniai modeliai leidžia detaliai ištirti procesą bei parinkti optimalias parametrų reikšmes. Matematinį modelį sudaro nestacionari lygtis, aprašanti difuzijos, konvekcijos ir cheminės reakcijos procesus. Vidurkinimo metodu uždavinys suvedamas į vienmatį modelį. Cheminės reakcijos greitis modeliuojamas panaudojant Aremijaus modelį. Pateikiami skaičiavimo eksperimento rezultatai.