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MODELLING OF NUCLEAR REACTORS DYNAMICS

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ABSTRACT

The point model of a nuclear reactor with delay in feedback line ,,power - reactivity" estimating the influence of six groups of delayed neutrons is investigated. A linear and non-linear analysis of the model is made.

1. INTRODUCTION

Let us take the point model of a nuclear reactor that was suggested in [1]. The equations of its dynamics are given by

$$\dot{N}(t) = r_N[1 + a(1 - \frac{C(t)}{C_0}) - \frac{N(t - h_N)}{N_0}]N(t), \tag{1.1}$$

$$\dot{C}(t) = r_c[\frac{N(t)}{N_0} - \frac{1}{C_0}\sum_{j=1}^{6} \alpha_j C(t - h_j)]C(t). \tag{1.2}$$

Here $N(t)$ is the density of neutrons at the time moment time t; N_0 is its steady state; r_N is the coefficient of linear growth of the density of neutrons; $C(t)$ is the summarized density of all delayed neutrons at the time t; C_0 is its steady meaning; r_c is the coefficient of linear growth of the density of delayed neutrons; $h_N > 0$ is the delay in of feedback line "power-reactivity"; $h_j > 0$ is the delay, meaning the generation time of delayed neutrons group j; $\alpha_j = \frac{\beta_j}{\beta}$ is the relative yield of delayed neutrons ($\sum_{j=1}^{6} \alpha_j = 1$); β_j is part of delayed neutrons of the j kind; β is the part of all delayed neutrons ($\beta = \sum_{j=1}^{6} \beta_j$); $a\,(-1 < a \leq 0)$ is the parameter regulating the power of the reactor.

As the delayed neutrons make up from 0.7% to 1.5% of the whole number of neutrons, so a will be considered as a small parameter.

2. LINEAR ANALYSIS

Let $a = 0$. Then the system (1.1) – (1.2) has two equilibrium states

$$N\,(t) \equiv 0, \quad C(t) \equiv 0,$$

and

$$N(t) \equiv N_0, \quad C(t) \equiv C_0.$$

It is known that the zero equilibrium state is non-stable. We shall analyse the system (1.1) – (1.2) in the neighbourhood of non-zero equilibrium state. After the substitution of

$$N(t) = N_0[1 + x(t)], \tag{2.1}$$

$$C(t) = C_0[1 + y(t)] \tag{2.2}$$

into equations (1.1) – (1.2), we get the equations

$$\dot{x}\,(t) + r_N[1 + x(t)]x(t - h_N) = 0, \tag{2.3}$$

$$\dot{y}\,(t) = r_c[x(t) - \sum_{j=1}^{6} \alpha_j y(t - h_j)][1 + y(t)]. \tag{2.4}$$

The linear parts of (2.3), (2.4) are given by

$$\dot{x}\,(t) = -r_N x(t - h_N), \tag{2.5}$$

$$\dot{y}\,(t) = r_c[x(t) - \sum_{j=1}^{6} \alpha_j y(t - h_j)]. \tag{2.6}$$

The characteristic equation of the system (2.5) – (2.6) is defined as

$$[1 + r_N \exp(-\lambda h_N)][\lambda + r_c \sum_{j=1}^{6} \alpha_j \exp(-\lambda h_j)] = 0. \tag{2.7}$$

The analysis of the roots of this equation splits into the investigation of two quasi-polynomial roots.

The disposition of the roots of the quasi-polynomial $P(\lambda)=\lambda+r_N\exp(-\lambda h_N)$ on the complex plane is well-known [2], but in order to determine the disposition of the roots of the quasi-polynomial

$$P(\lambda)=\lambda+r_c\sum_{j=1}^{6}\alpha_j\exp(-\lambda h_j) \tag{2.8}$$

on the complex plane we shall do a special research.

Introducing additional parameter p, we shall analyse the roots of the quasi-polynomial

$$P_p(\lambda)=\lambda+p+r_c\sum_{j=1}^{6}\alpha_j\exp(-\lambda h_j) \tag{2.9}$$

by the D – decomposition method [5]. If $\lambda=0$, then

$$p+r_c=0. \tag{2.10}$$

Line (2.10) is one of D – decomposition curves on the plane. Let $\lambda=i\sigma$. Then we get from (2.9) the following parameter equations of the other D – decomposition curves:

$$r_c=\frac{\sigma}{\sum\limits_{j=1}^{6}\alpha_j\sin\sigma h_j}, \tag{2.11}$$

$$p=-\frac{\sigma\sum\limits_{j=1}^{6}\alpha_j\cos\sigma h_j}{\sum\limits_{j=1}^{6}\alpha_j\sin\sigma h_j}=-r_c\sum_{j=1}^{6}\alpha_j\cos\sigma h_j. \tag{2.12}$$

In the case of $\sigma\to 0$, we find the coordinates of the recurrence point (p_0,z_0) of the curves (2.10) and (2.11) – (2.12)

$$(p_0;r_0)=\left(-\frac{1}{\sum\limits_{j=1}^{6}\alpha_j h_j};\frac{1}{\sum\limits_{j=1}^{6}\alpha_j h_j}\right). \tag{2.13}$$

D – decomposition of parametres p and r_c on the plane is given in fig. 1. The values of α_j and h_j are taken from table 1 [3] . Two roots with positive real parts appear in the region D_2.

Table 1.
D-decomposition.

Fuel	j	$T_{1/2} = h_j$ s	α_j
	1	55.72	0.033
	2	22.72	0.219
^{235}U	3	6.22	0.196
	4	2.30	0.395
	5	0.61	0.115
	6	0.23	0.042

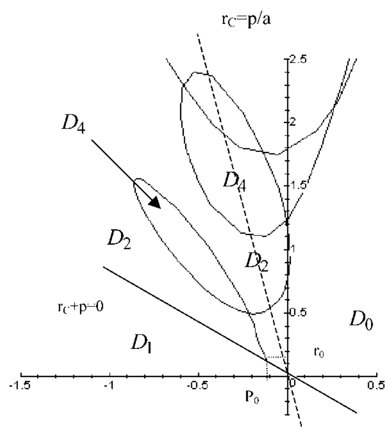

Figure 1. D – decomposition.

3. NONLINEAR ANALYSIS $(A = 0)$

3.1. The reason of oscillation - the feedback line

We take differential equations(2.3) – (2.4) If $\varepsilon = r_N h_N - \frac{\pi}{2}$ is a sufficiently small positive variable, then equation (2.3) has a stable periodical solution [2]

$$x(t) = \xi x_1(t) + \xi^2 x_2(t) + \cdots, \tag{3.1}$$

where

$$x_1(t) = \cos\sigma t,\ x_2(t) = \frac{1}{10}(\sin 2\sigma t + 2\cos 2\sigma t),$$
$$\sigma\left(1 + \frac{c_2}{b_2}\varepsilon + \cdots\right) = \frac{\pi}{2h_N},\quad \xi = \sqrt{\frac{\varepsilon}{b_2}},$$
$$c_2 = \frac{1}{10\pi},\ b_2 = \frac{3\pi - 2}{40}.$$

Theorem 3.1. *If* $r_c \sum_{j=1}^{6} \alpha_j h_j \leq 1$, *then the roots of quasi-polynomial (2.8) satisfy the inequality* $\operatorname{Re}\lambda < 0$ *and the differential equation (2.4) has a* *stable periodical solution.*

Conclusion. *We can calculate the stable periodical solution of the equation (2.4) using the formula*

$$y(t) = \xi y_1(t) + \xi^2 y_2(t) + \cdots, \tag{3.2}$$

where functions $y_j(t)$ *are found using the method of undetermined coefficients.*

Thus

$$\dot{y}_1(t) + r_c \sum_{j=1}^{6} \alpha_j y_1(t - h_j) = r_c x_1(t), \tag{3.3}$$

$$\dot{y}_2(t) + r_c \sum_{j=1}^{6} \alpha_j y_2(t - h_j) = r_c x_2(t) + y_1(t)\, \dot{y}_1(t). \tag{3.4}$$

We get from the equation (3.3)

$$y_1(t) = \frac{r_c}{|P(i\sigma)|^2}[\operatorname{Im} P(i\sigma)\sin\sigma t + \operatorname{Re} P(i\sigma)\cos\sigma t]. \tag{3.5}$$

We get from the equation (3.4) that

$$y_2(t) = A\sin 2\sigma t + B\cos 2\sigma t, \tag{3.6}$$

where

$$A = \frac{1}{|P(2i\sigma)|^2}[W_1 \operatorname{Re} P(2i\sigma) + W_2 \operatorname{Im} P(2i\sigma)], \tag{3.7}$$

$$B = \frac{1}{|P(2i\sigma)|^2}[W_2 \operatorname{Re} P(2i\sigma) - W_1 \operatorname{Im} P(2i\sigma)], \tag{3.8}$$

$$W_1 = \frac{1}{10} r_c + \frac{\sigma r_c^2}{2\,|P(i\sigma)|^2}[\operatorname{Im}^2 P(i\sigma) - \operatorname{Re}^2 P(i\sigma)], \tag{3.9}$$

$$W_2 = \frac{1}{5} r_c + \frac{\sigma r_c^2}{|P(i\sigma)|^2}[\operatorname{Re} P(i\sigma) \operatorname{Im} P(i\sigma)]. \tag{3.10}$$

Therefore, differential equations (1.1) and (1.2) have the stable periodical solutions

$$N(t) = N_0[1 + \xi \cos\frac{\pi}{2h_N}\tau + \xi^2 x_2(\tau) + o(\xi^3)], \tag{3.11}$$

$$C(t) = C_0[1 + \xi y_1(\tau) + \xi^2 y_2(\tau) + o(\xi^3)], \tag{3.12}$$

where functions $x_2(\tau)$, $y_1(\tau)$, $y_2(\tau)$ and variable ξ, b_2, c_2 are defined by formulae (3.1),(3.5) – (3.10), withh $\sigma = \frac{\pi}{2h_N}$, $\tau = \frac{t}{1 + c_2\xi^2}$.

3.2. Oscillations of two frequencies

We shall analyse the system

$$\dot{N}(t) = r_N[1 - \frac{N(t-h_N)}{N_0}]N(t), \tag{3.13}$$

$$\dot{C}(t) = r_c[\frac{N(t)}{N_0} - \frac{1}{C_0}\sum_{j=1}^{6}\alpha_j C(t-h_j)]C(t). \tag{3.14}$$

Let $r_N = \frac{\pi}{2h_N} + \varepsilon$, $r_c = r_c^* + \mu$, parameters ε, μ are assumed to be small, and $r_c^* = \frac{\sigma_*}{\sum_{j=1}^{6}\alpha_j \sin\sigma_* h_j}$, where σ_* is the unique root of the equation $\sum_{j=1}^{6}\alpha_j\cos\sigma h_j = 0$ belonging to the interval $(0, \frac{\pi}{2h_N})$.

After substitution (2.1) – (2.2) into equations (3.13 – (3.14) we get differential equations

$$\dot{x}(t) + (\frac{\pi}{2h_N} + \varepsilon)x(t-h_N)[1 + x(t)] = 0, \tag{3.15}$$

$$\dot{y}(t) + (r_c^* + \mu)[\sum_{j=1}^{6}\alpha_j y(t-h_j) - x(t)][1 + y(t)] = 0. \tag{3.16}$$

When $0 < r_N - \frac{\pi}{2h_N} = \varepsilon \leq 1$, the differential equation (3.15) has a stable periodical solution [2]

$$x(t) = \xi\cos\frac{\pi}{2h_N}\nu_1 t + o(\xi^2), \tag{3.17}$$

where

$$\xi = \sqrt{\frac{h_N\varepsilon}{b_2}},\ \nu_1 = 1 - c_2\xi^2; \tag{3.18}$$

$$b_2 = \frac{3\pi - 2}{40},\ c_2 = \frac{1}{10\pi}. \tag{3.19}$$

If $\varepsilon = \mu = 0$, then the characteristic equation (2.7) of the linear part of differential equations (3.15) – (3.16) has two pairs of purely imaginary roots $\pm i\frac{\pi}{2h_N}$, $\pm i\sigma_*$ and the real parts of the other roots are negative. Then the system of differential equations (3.15) – (3.16) under certain conditions has a stable solution of two frequencies. The asymptotic expression of this solution is too complicated, but the formed statements easily tell the peculiarities of the numerical expression of this solution.

In this case the oscillation of two frequencies is caused by the perturbations in feedback line together with the influence of delayed neutrons.

3.3. The reason of oscillation given by the influence of delayed neutrons

Let assume that $r_N h_N < \frac{\pi}{2}$, but the characteristic quasi-polynomial has one pair of purely imaginary roots $\pm i\sigma_*$, and real parts of other roots are negative. This analysis is done in the neighbourhood of the equilibrium state $C(t) = C_0$.

Let us analyse the equation (3.16)

It is clear that $x(t) \to 0$, when $t \to \infty$. Our problem leads to finding a periodical solution of the equation (3.16). It is not difficult to get the asymptotic expression of this solution (see [2]).

4. NONLINEAR ANALYSIS ($A \neq 0$)

The earlier analysed reasons of the appearance of periodical solutions at $a = 0$ let us localize clearly enough the parameters of the analysed model. At the same time nonlinear analysis should not cause great problems in general.

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BRANDUOLINIO REAKTORIAUS DINAMIKOS MATEMATINIS MODELIAVIMAS

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Darbe sudarytas branduolinio reaktoriaus matematinis modelis, kuris yra aprašomas dviejų paprastųjų diferencialinių lygčių su vėluojančiu argumentu sistema. Tiriamas šio modelio sprendinio stabilumas. Nurodytos sąlygos, kada atsiranda osciliuojantys sprendiniai, ištirtos tokios bifurkacijos priežastys. Matematiniai teiginiai yra įrodomi panaudojant D - suskaidymo metodą.